Macroeconomic Forecasting With Mixed-Frequency Data: Forecasting Output Growth in the United States

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Many macroeconomic series, such as U.S. real output growth, are sampled quarterly, although potentially useful predictors are often observed at a higher frequency. We look at whether a mixed data-frequency sampling (MIDAS) approach can improve forecasts of output growth. The MIDAS specification used in the comparison uses a novel way of including an autoregressive term. We find that the use of monthly data on the current quarter leads to significant improvement in forecasting current and next quarter output growth, and that MIDAS is an effective way to exploit monthly data compared with alternative methods.

KEY WORDS: Forecasting; Mixed-frequency data; U.S. output growth.

1. INTRODUCTION

The unavailability of key macroeconomic variables, such as GDP (or GNP), at frequencies higher than quarterly has led to the specification of many macroeconometric models on quarterly data. We consider the usefulness of information available at higher frequencies, such as monthly data, for forecasting output growth. A range of leading and coincident indicator variables at a monthly frequency are available. If all of the variables in the model must to be sampled at the same frequency, then data available at the monthly frequency must be converted to the quarterly frequency by, for example, averaging the months (or taking the last month in the quarter), and information on the first month (or first two months) of the quarter being forecast is discarded.

In this article we explore whether the mixed data sampling (MIDAS) approach of Ghysels, Santa-Clara, and Valkanov (2004) and Ghysels, Sinko, and Valkanov (2006b) can be successfully adapted to the modeling and forecasting of a key U.S. macroeconomic variable: U.S. output growth. MIDAS allows sampling of the regressand and regressors at different frequencies. Typically, the regressand is sampled at the lower frequency. With few exceptions, MIDAS has been used for high-frequency financial data (see, e.g., Ghysels et al. 2006a), although Ghysels and Wright (2006) looked at survey-based macroeconomic forecasts. We compare the results of using MIDAS to forecast output growth with two other approaches that exploit monthly data. The first of these approaches is an extension to the models used by Koenig, Dolmas, and Piger (2003), and the second is a two-step procedure that generates forecasts of missing monthly indicator values, which are then averaged to generate quarterly observations. The MIDAS specification used in the comparison uses a novel way of including an autoregressive (AR) term.

Looking ahead to the results, we find that the use of within-quarter information on monthly indicators can result in marked reductions in RMSE compared with quarterly-frequency AR or AR distributed-lag (ADL) models. Within the set of models that use monthly information, MIDAS fares well across the set of indicators that we consider. Coupled with their flexibility and ease of use, MIDAS models provide an attractive way of exploiting the information in monthly indicators.

These findings are based on a recursive out-of-sample forecasting exercise that uses "conventional" real-time data. As explained by Koenig et al. (2003), the use of real-time data is clearly preferable to the use of final-revised data for estimation and evaluation purposes, because out-of-sample forecasting exercises based on final-revised data may exaggerate the predictive power of explanatory variables relative to what actually could have been achieved at the time (Diebold and Rudebusch 1991; Orphanides 2001; Orphanides and van Norden 2005; Faust, Rogers, and Wright 2003). Koenig et al. (2003) noted that the way in which real-time data are conventionally used in forecast comparison exercises is based on the use of end-of-sample vintage data. They argued that this approach to real-time estimation and forecasting may be suboptimal, that "at every date within a sample, right-side variables ought to be the most up-to-date estimates available *at that time*" (Koenig et al. 2003, p. 618, described as strategy 1 on p. 619). We find that Koenig et al.'s suggestion to use real-time vintage data to estimate forecasting models improves forecast accuracy, and that our main conclusions are unchanged: The inclusion of monthly data in the forecasting problem can dramatically reduce RMSE, and MIDAS is an effective way of incorporating monthly data.

The rest of the article is organized follows. Section 2 briefly reviews the MIDAS approach of Ghysels et al. (2004, 2006b) and proposes an extension that facilitates the application of MIDAS to macroeconomic data, namely the inclusion of an AR component. It also describes the two other approaches to using monthly indicator data to generate forecasts of quarterly output growth. Section 3 contains the out-of-sample forecast comparison exercise, split into five parts. Section 3.1 describes our use of monthly and quarterly vintages to estimate and forecast in real time. Section 3.2 compares the MIDAS–AR forecasts against the quarterly AR model forecasts using both a

Journal of Business & Economic Statistics
October 2008, Vol. 26, No. 4
DOI 10.1198/073500108000000015

conventional real-time vintage data approach and final-revised data, to see whether the incorporation of monthly indicator data in the forecasting model results in significant improvements in forecast accuracy. Section 3.3 compares alternative ways of incorporating monthly data into the forecasting problem. Section 3.4 investigates the use of real-time vintage data in the forecast comparison exercise. Finally, Section 3.5 compares the multiple-indicator model forecasts with combining individual model forecasts. Section 4 offers some concluding remarks.

2. THE MIDAS REGRESSION APPROACH

The MIDAS models of Ghysels et al. (2004, 2006b) are closely related to distributed lag models (see, e.g., Dhrymes 1971; Sims 1974). The response of the dependent variable to the higher-frequency explanatory variables is modeled using highly parsimonious distributed lag polynomials, as a way of preventing the proliferation of parameters that might otherwise result and of side-stepping difficult issues related to lag-order selection. Parameter proliferation could be especially important in financial applications, where, say, daily volatility is related to 5-minute interval intraday data (so that a day's worth of observations amounts to 288 data points), but parsimony also is likely to be important in typical macroeconomic applications, where quarterly data are related to monthly data, given the much smaller numbers of observations typically available. Modeling the coefficients on the lagged explanatory variables as a distributed lag function allows for long lags, with the need to estimate only a small number of parameters.

The basic MIDAS model for a single explanatory variable, and h-step-ahead forecasting, is given by

$$y_t = \beta_0 + \beta_1 B\big(L^{1/m};\boldsymbol{\theta}\big)x_{t-h}^{(m)} + \varepsilon_t, \tag{1}$$

where $B(L^{1/m};\boldsymbol{\theta}) = \sum_{k=1}^{K} b(k;\boldsymbol{\theta})L^{(k-1)/m}$ and $L^{s/m}x_{t-1}^{(m)} = x_{t-1-s/m}^{(m)}$. Here t indexes the basic time unit (in our case, quarters), m is the higher sampling frequency ($m = 3$ when x is monthly and y is quarterly), and, as shown, $L^{1/m}$ operates at the higher frequency. All of the parameters of the MIDAS model depend on the horizon h (although this is suppressed in the notation), and forecasts are computed directly without the need for forecasts of explanatory variables. An "exponential Almon lag" function (Ghysels et al. 2004, 2006b) parameterizes $b(k;\boldsymbol{\theta})$ as

$$b(k;\boldsymbol{\theta}) = \frac{\exp(\theta_1 k + \theta_2 k^2)}{\sum_{k=1}^{K}\exp(\theta_1 k + \theta_2 k^2)}. \tag{2}$$

Because macroeconomic forecasts often are produced a number of times during each quarter, monthly data on relevant indicators for the quarter being forecast sometimes will be available. For example, the staff of the Board of Governors of the Federal Reserve prepare forecasts several times each quarter for the meetings of the Open Market Committee (see Karamouzis and Lombra 1989; Joutz and Stekler 2000). For illustrative purposes, suppose that the value of x in the first month of the quarter is known. The MIDAS framework can exploit these data by simply specifying the regression model as

$$y_t = \beta_0 + \beta_1 B\big(L^{1/3};\boldsymbol{\theta}\big)x_{t-2/3}^{(3)} + \varepsilon_t,$$

where $h = 2/3$ signifies that $1/3$ of the information on the current quarter is used. Forecasts with $h = 1/3$ also are possible using information on the first two months of the quarter being forecast. Thus the MIDAS model can incorporate within-quarter monthly observations on the indicator variable in a simple fashion.

2.1 Autoregressive Structure

Models to forecast U.S. output growth often include AR terms, as in the ADL models of Stock and Watson (2003). Including AR dynamics in models that sample the explanatory variables at a higher frequency clearly is desirable. As noted by Ghysels et al. (2006b), this is not straightforward; consider simply adding a lower-frequency lag of y, y_{t-1}, to (1) for one-step-ahead forecasting, to give

$$y_t = \beta_0 + \lambda y_{t-1} + \beta_1 B\big(L^{1/3};\boldsymbol{\theta}\big)x_{t-1}^{(3)} + \varepsilon_t.$$

This strategy generally is not appropriate, because from writing the model as

$$y_t = \beta_0(1-\lambda)^{-1} + \beta_1(1-\lambda L)^{-1}B\big(L^{1/3};\boldsymbol{\theta}\big)x_{t-1}^{(3)} + \widetilde{\varepsilon}_t,$$

where $\widetilde{\varepsilon}_t = (1-\lambda L)^{-1}\varepsilon_t$, it is apparent that the polynomial on $x_{t-1}^{(3)}$ is the product of a polynomial in $L^{1/3}$, $B(L^{1/3};\boldsymbol{\theta})$, and a polynomial in L, $\sum \lambda^j L^j$. This mixture generates a "seasonal" response of y to $x^{(3)}$ irrespective of whether or not $x^{(3)}$ displays a seasonal pattern.

Our suggested solution is simply to introduce AR dynamics in y_t as a common factor (see, e.g., Hendry and Mizon 1978),

$$y_t = \beta_0 + \lambda y_{t-1} + \beta_1 B\big(L^{1/3};\boldsymbol{\theta}\big)(1-\lambda L)x_{t-1}^{(3)} + \varepsilon_t, \tag{3}$$

so that the response of y to $x^{(3)}$ remains nonseasonal. A multistep analog can be written as

$$y_t = \lambda y_{t-d} + \beta_1 + \beta_2 B\big(L^{1/3},\boldsymbol{\theta}\big)(1-\lambda L^d)x_{t-h}^{(3)} + \epsilon_t, \tag{4}$$

which we term the MIDAS–AR. When the horizon h is an integer, $d = h$, as in (3), where $d = h = 1$. When information is available on the indicator in the current quarter (say, the first two months are known), $h = 1/3$, whereas $d = 1$.

The referenced literature establishes that nonlinear least squares is a consistent estimator for the standard MIDAS. To estimate the MIDAS–AR model, we take the residuals $(\widehat{\varepsilon}_t)$ of the standard MIDAS, and estimate an initial value for λ (say $\hat{\lambda}_0$) from $\hat{\lambda}_0 = (\sum \widehat{\varepsilon}_{t-h}^2)^{-1}\sum \widehat{\varepsilon}_t\widehat{\varepsilon}_{t-h}$. We then construct $y_t^* = y_t - \hat{\lambda}_0 y_{t-d}$ and $x_{t-h}^{*(3)} = x_{t-h}^{(3)} - \hat{\lambda}_0 x_{t-h-d}^{(3)}$, and obtain the estimator $\hat{\theta}_1$ by applying nonlinear least squares to

$$y_t^* = \beta_1 + \beta_2 B\big(L^{1/3},\boldsymbol{\theta}\big)x_{t-h}^{*(3)} + \varepsilon_t.$$

We obtain a new value of λ—$\hat{\lambda}_1$—from the residuals of this regression. Then, using $\hat{\lambda}_1$ and $\hat{\theta}_1$ as initial values, we run Broyden–Fletcher–Goldfarg–Shanno (BFGS) to get the estimates $\hat{\lambda}$ and $\hat{\boldsymbol{\theta}}$ that minimize the sum of squared residuals. We carry out the computations using the constrained maximum likelihood package of Gauss 5 (CML 2.0) and selecting the BFGS algorithm. The restrictions imposed in the estimation are that $\theta_1 \le 300$ and $\theta_2 < 0$, and we experiment with a number of initial values for $\boldsymbol{\theta}$ to counter any dependence of the optimization routine on the initial values.

2.2 Combining Indicators

An M–MIDAS–AR model that combines the information of nl monthly leading indicators to predict h-steps-ahead output growth would be written as

$$y_t = \lambda y_{t-d} + \beta_0 + \sum_{i=1}^{nl} \beta_{1i} B_i\big(L^{1/m}; \boldsymbol{\theta}_i\big)(1 - \lambda L^d) x_{i,t-h}^{(m)} + \varepsilon_t, \tag{5}$$

where the component indicators are indexed by i and $m = 3$. Each leading indicator requires the estimation of only two parameters to describe the lag structure ($\boldsymbol{\theta}_i$) and one parameter to weight their impact on y_t (β_{1i}). Because the number of parameters required for each additional leading indicator is small, a good forecast performance from MIDAS might be anticipated when multiple indicators are included in the forecasting model, relative to other models in which the larger number of indicators is accommodated at the cost of many more parameters to be estimated.

2.3 Alternative Methods of Exploiting Monthly Indicator Data

In addition to MIDAS, two other methods are used in the empirical forecast comparisons. The forecasting models used by Koenig et al. (2003) regress quarterly changes in real GDP on a constant and five lagged monthly changes in the monthly indicator variables. Their forecasting models are similar to the MIDAS approach, except that the coefficients on the right-side explanatory variables are estimated unrestrictedly, rather than using a restricted distributed lag function, and there are no AR terms, compared with our MIDAS–AR. Koenig et al. (2003) calculate forecasts only when the values of the indicators for all the months in the quarter being forecast are known. We call these models *mixed-frequency distributed lag* (MF–DL) models and use them to generate forecasts for a number of monthly horizons (in addition to Koenig et al.'s $h = 0$) when only partial monthly information is available on the quarter being forecast (corresponding to $h = 1/3$ and $h = 2/3$).

The second method is to use a vector autoregression (VAR) comprising of the monthly indicator variables to provide forecasts of the missing monthly values, which are then aggregated to provide estimates of the quarterly values of the indicators. This method resembles the "bridge equation" approach popular with the Central Banks (see, e.g., Rünstler and Sédillot 2003; Zheng and Rossiter 2006). As an example, suppose that we have data only on the first month of quarter t; that is, $x_{t-2/3}^{(3)}$ is known, but $x_t^{(3)}$ and $x_{t-1/3}^{(3)}$ are not yet known. Forecasts of $\{x_t^{(3)}, x_{t-1/3}^{(3)}\}$ are obtained from the VAR, $\{\widehat{x}_t^{(3)}, \widehat{x}_{t-1/3}^{(3)}\}$, and the quarterly estimate of x_t is constructed as $\widehat{x}_t = \frac{1}{3}(\widehat{x}_t^{(3)} + \widehat{x}_{t-1/3}^{(3)} + x_{t-2/3}^{(3)})$, which is used in the quarterly-frequency ADL model to forecast y_t. We refer to the approach that uses forecasts of missing monthly observations to augment a quarterly-frequency ADL as the ADL–F. When the forecast horizon is an integer number of quarters, forecasts of the monthly indicator are not required, and ADL–F corresponds to the standard quarterly-frequency ADL. When using single-indicator models, we use an AR rather than a VAR to compute forecasts for the missing monthly values, so that the same indicator information is available for all models.

There are other ways of using monthly information; for example, Miller and Chin (1996) proposed combining the forecasts from a monthly model with forecasts from a quarterly model. There also are factor model approaches that make use of mixed-frequency data, such as the model of Schumacher and Breitung (2006), which adapt single-frequency factor models (see Boivin and Ng 2005 for a review). However, for the purpose of evaluating the accuracy of MIDAS models, we chose MF–DL and ADL–F because they are simple and popular methods when a relatively small number of indicators is available.

3. EMPIRICAL FORECASTING COMPARISONS

The relative forecast performance of the models is assessed by comparing RMSEs in a recursive forecasting exercise. Because we also exploit monthly vintages of the indicators while forecasting in real time, we first describe how end-of-sample vintage data and real-time vintage data are used for model estimation and forecasting in this context. To highlight the principal findings, we then present the forecast comparisons in five sections. The first of these compares the MIDAS–AR forecasts against the quarterly AR(1) and ADL forecasts. The number of lags of the indicator in the ADL is selected using the Schwarz criterion (SIC), setting the maximum to 5. Section 3.2 compares the relative performances when end-of-sample vintage data are used and when final-revised data are used to see whether the predictive content of monthly data (through the MIDAS–AR) is sensitive to this issue. (Recall that in Sec. 1 we referenced a number of studies where this issue was key.) Section 3.3 compares the MIDAS–AR against the two alternative ways of using monthly data (MF–DL and ADL–F) based on a conventional real-time data forecasting exercise. Section 3.4 investigates whether the result of the forecast comparison changes with the use of real-time vintage data. Finally, Section 3.5 compares the multiple-indicator model forecasts with combinations of individual model forecasts, motivated by the vast literature attesting to the usefulness of forecast combination (see the recent reviews of Diebold and Lopez 1996; Newbold and Harvey 2002; Timmermann 2006; Clements and Harvey 2008).

3.1 Use of Vintage Data in the Real-Time Forecasting Exercises

Our real-time data consist of quarterly vintages of output growth and monthly vintages of the indicators, obtained from the Philadelphia Fed (see Croushore and Stark 2001). We consider three monthly indicators: industrial production (IP), employment (EMP; payroll, nonfarm), and capacity utilization (CU). The first two indicators are components of the Conference Board Coincident Index; and all three series, as well as real output, were downloaded from *http://www.phil.frb.org*. Before using the indicator data set for forecasting, we construct approximate monthly growth rates by taking the first difference of the log of each series. Real output growth is the quarterly difference of the log of output. The quarterly real-time data sets of the Philadelphia Fed record the data available on the 15th of the middle month of a quarter, so that for output, the data sets contain the Bureau of Economics Analysis (BEA) "advance" estimates for the latest quarter (the previous quarter) as well as revised data for earlier quarters. In principle, it may be possible to incorporate the different releases of output data that are made during the course of a quarter into the forecast comparisons, but data availability prevented us from considering this option.

The exercise consists of forecasting output growth in the quarters 1985:Q2–2005:Q1. For each of these quarters, we generate forecasts with horizons from $h = 0$ up to 2 quarters, with monthly steps, $h = 1/3$, $h = 2/3$, and so on in the case of the forecasting models that make use of monthly indicator information, as described more fully herein. The model estimation sample begins in 1959:Q1. The monthly data consist of monthly vintages of the indicators from 1985:M1–2005:M1. For expositional purposes, let y_{t+1} denote current quarter output growth and y_{t+2} denote next quarter output growth.

The timing of the release of official data on output growth and the monthly indicators is as follows. The vintage data for output growth for quarter $t + 1$ contain data up to quarter t. Before data on output growth for $t + 1$ become available in the $t + 2$ quarterly vintage, we will have data on x up to $t + 1/3$ from the $t + 2/3$ monthly vintage, data on x up to $t + 2/3$ from the $t + 1$ monthly vintage, and data on x up to $t + 1$ from the $t + 4/3$ monthly vintage. (We suppress the $^{(3)}$ superscript on x—it is implicit that x is recorded at the monthly frequency and y is quarterly.) The timing of these three monthly releases relative to the release of the quarterly data gives rise to forecast horizons of $h = 2/3$, $1/3$, and 0. We adopt the notation that $y_{\tau;v}$ denotes the value of y in period τ in the vintage v data set (and similarly for x, where the monthly frequency gives rise to τ and v being measured as fractions of quarters). Following Koenig et al. (2003) and others, our aim is to forecast the final output growth numbers, where the "final" data are taken to be the latest vintage data to which we have access (2005:Q2), which we denote by T, so that $y_{t+1;T}$ denotes the estimate of the actual value of y in $t + 1$ in the final vintage data.

We use two ways of exploiting the real-time data set for forecasting. We first use the "traditional" or end-of-sample vintage data approach to real-time forecasting, then outline the proposal of Koenig et al. (2003) to use real-time vintage data.

3.1.1 End-of-Sample Vintage Data Real-Time Forecasting With Monthly Data. At each forecast origin, the models are estimated and the forecasts computed using the data contained in the most recent data sets available at that time. For example, for a one-step-ahead forecast of $y_{t+1;T}$ from an AR(1), we regress $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-1;t+1}$ (and a constant), where $\mathbf{y}_{t;t+1} = [y_{2;t+1}, \dots, y_{t;t+1}]'$ and $\mathbf{y}_{t-1;t+1} = [y_{1;t+1}, \dots, y_{t-1;t+1}]'$, and use the estimated model coefficients and right-side y-value of $y_{t;t+1}$ to compute the forecast $\hat{y}_{t+1;T}$. When we have monthly vintages of indicator data, the models that use this information (MIDAS–AR, MF–DL, and ADL–F) provide four forecasts of current quarter output growth ($y_{t+1;T}$), depending on where we are in the current quarter, as described earlier. Suppose that we have data on the indicator for all of the months in the quarter, but not for the current quarter value of output growth: we designate this a zero-horizon forecast (or "nowcast"). Using MIDAS as an example, we regress $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-1;t+1}$ and $B(L^{1/3}; \boldsymbol{\theta})\mathbf{x}_{t;t+4/3}$ (and a constant), where $\mathbf{x}_{t;t+4/3} = [x_{2;t+4/3}, \dots, x_{t-1;t+4/3}, x_{t;t+4/3}]'$, and the monthly lags of this vector are all from the $t + 4/3$ monthly vintage. Then, using these estimates and the last available information on y and x at vintages $t + 1$ and $t + 4/3$ (namely $y_{t;t+1}$ and $\mathbf{x}_{t+1;t+4/3}$), we obtain the forecasts.

When indicator information is available on the first two months of the current quarter, the $h = 1/3$ forecast of the current quarter and the $h = 4/3$ forecast of the next quarter ($t + 2$) are constructed as follows. For $h = 1/3$, the MIDAS–AR regresses $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-1;t+1}$ and $B(L^{1/3}; \boldsymbol{\theta})\mathbf{x}_{t-1/3;t+1}$ (and a constant), where $\mathbf{x}_{t-1/3;t+1} = [x_{2-1/3;t+1}, \dots, x_{t-1/3;t+1}]'$ with $d = 1$. The forecast is conditioned on $y_{t;t+1}$ and $x_{t+2/3;t+1}$. For $h = 4/3$, the forecast is generated from a regression of $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-2;t+1}$ and $B(L^{1/3}; \boldsymbol{\theta})\mathbf{x}_{t-4/3;t+1}$, again using the last values in the vintage data sets to compute the forecasts. When information only on the first month in the quarter is available, the forecasts horizons are equal to $2/3$ and $5/3$. We again use data on y from the $t + 1$ vintage data, but take the data on the indicator from the $t + 2/3$ vintage data. Finally, we obtain an $h = 1$ forecast of y_{t+1} and an $h = 2$ forecast of y_{t+2} when the latest data are $\mathbf{y}_{t;t+1}$ and $\mathbf{x}_{t;t+1/3}$.

This is the traditional way ("end-of-sample") to conduct a real-time forecasting exercise, adapted to incorporate monthly vintages of monthly indicator data.

3.1.2 A Real-Time Vintage Data Approach to Real-Time Forecasting With Monthly Data. The difference between the end-of-sample and real-time vintage approaches can be seen most easily by considering forecasting $y_{t+1;T}$ with a single x, for the case where $h = 0$. The end-of-sample vintage approach estimates

$$y_{s;t+1} = x_{s;t+4/3}\beta + v_s \tag{6}$$

on $s = 1, \dots, t$, where $t + 1 \le T$, and forecasts y_{t+1} as $\hat{y}_{t+1} = x_{t+1;t+4/3}\hat{\beta}$. Koenig et al. (2003) showed that under general conditions, $\hat{\beta}$ will be an inconsistent estimator of β_0, where β_0 relates the true value of y_t to x_t: $y_t = x_t\beta_0 + \varepsilon_t$, because $\hat{\beta}$ will reflect in part the nature of the joint revision process rather than cleanly capturing the forecasting relationship between y_t and x_t. A consistent estimator of β_0 can be obtained from estimating

$$y_{s;s+1} = x_{s;s+1/3}\beta_0 + \epsilon_s \tag{7}$$

on $s = 1, \dots, t$, that is, using real-time vintage data. Forecasts are computed as $\hat{y}_{t+1} = x_{t+1;t+4/3}\hat{\beta}_0$, as in the end-of-sample vintage approach.

The approaches exemplified by (6) and (7) condition on the same information in the estimated model to generate forecasts but, as is evident, differ in the way in which the estimation sample is constructed. Because we have monthly and quarterly vintages of data and wish to calculate forecasts with monthly horizons, some care is required when implementing the real-time vintage scheme of (7) in our context. First, consider the AR(1) benchmark, against which we judge the accuracy of the forecasts from the models with monthly indicators. For the AR(1), the left-side and right-side vectors of observations are given by $[y_{2;2}, \dots, y_{t-2;t-1}, y_{t-1;t}, y_{t;t+1}]'$ and $[y_{1;2}, \dots, y_{t-3;t-1}, y_{t-2;t}, y_{t-1;t+1}]'$. Suppose that we have a model with two lags of x. If all of the months for the current quarter are available, $h = 0$, then the logic of the real-time vintage approach suggests augmenting the right-side AR(1) data vector with the vectors of observations on x given by $[\dots, x_{t-1;t-2/3}, x_{t;t+1/3}]'$ and $[\dots, x_{t-4/3;t-2/3}, x_{t-1/3;t+1/3}]'$. Note that the sequence of vintages does not change with the inclusion of lag values, implying that some data used in the estimation have been partially revised. The last available data are used to compute the forecasts $(y_{t;t+1}, x_{t+1;t+4/3}, x_{t+2/3;t+4/3})$, as in the end-of-sample approach. Although our models use longer lags that of x than the two lags we consider, no new issues arise.

For $h = 1/3$, the two x-vectors used in estimation are given by $[\dots, x_{t-4/3;t-1}, x_{t-1/3;t}]'$ and $[\dots, x_{t-5/3;t-1}, x_{t-2/3;t}]'$, and for $h = 4/3$, $[\dots, x_{t-7/3;t-1}, x_{t-4/3;t}]'$ and $[\dots, x_{t-8/3;t-1}, x_{t-5/3;t}]'$. For both horizons, the estimated models are used to generate forecasts using $(x_{t+2/3;t+1}, x_{t+1/3;t+1})$. When only first-month information is available, we build the right-side vectors in a similar fashion; thus, for example, for $h = 2/3$, the two x-vectors are given by $[\dots, x_{t-1-2/3;t-1}, x_{t-2/3;t}]'$ and $[\dots, x_{t-2;t-1}, x_{t-1;t}]'$. Finally, for $h = 1$, the x-vectors are $[\dots, x_{t-2;t-5/3}, x_{t-1;t-2/3}]'$ and $[\dots, x_{t-7/3;t-5/3}, x_{t-4/3;t-2/3}]'$.

3.2 Does Monthly Indicator Information Help? MIDAS–AR versus AR and ADL

The first set of results uses end-of-sample vintage data in a real-time forecasting exercise. However, to establish a benchmark, we first discuss results obtained using final-revised vintage data throughout. We compare forecasts of the MIDAS–AR with an AR and an ADL, to determine whether monthly indicator information improves forecast accuracy. Table 1 gives the ratios of the RMSEs of the MIDAS–AR against the AR and ADL models. The table shows sizeable reductions in RMSE using MIDAS when monthly data are available on the current quarter; these are of the order of 20% when industrial production is the indicator for "nowcasts" and $h = 1/3$ horizons (compared with the quarterly-frequency ADL). The RMSEs are calculated for output growth rates calculated as 100 times the quarterly difference of the natural log of GDP. So taking the MIDAS RMSE to be .4 for illustrative purposes, a RMSE ratio of the MIDAS to the ADL of .8 translates into respective RMSEs at annual rates of 1.6% and 2.0%, set against an average annual growth rate of 3.2% over the period.

Sizeable gains also are achieved when the indicator is capacity utilization for these horizons, whereas the differences between MIDAS and the AR and ADL using employment are smaller. We also report average ratios across horizons, which confirm that MIDAS generally is no better (and often is worse) when no monthly information on the quarter is available.

In passing, we note that a comparison of the quarterly frequency ADL and AR models (with ADL/AR ratios obtained by dividing the MIDAS/AR ratio by the MIDAS/ADL ratio for each indicator in Table 1) presents a less upbeat picture of the usefulness of the indicators. Relative to an AR, the quarterly frequency ADL enhances accuracy only when employment is the indicator.

Table 1. Comparing the forecasting performance of MIDAS–AR with the AR and ADL using real-time end-of-sample vintage data and final-revised data

Horizon (h)		Industrial production (IP)			Employment (EMP)			Capacity utilization (CU)		
			Ratio to			Ratio to			Ratio to	
MIDAS	AR, ADL	MIDAS–AR (RMSE)	AR	ADL	MIDAS–AR (RMSE)	AR	ADL	MIDAS–AR (RMSE)	AR	ADL
Using end-of-sample vintage data										
0	1	.444	**.902**	**.877**	.514	1.042	1.014	.452	**.916**	**.895**
1/3	1	.432	**.876**	**.851**	.487	.989	.962	.439	**.890**	**.870**
2/3	1	.516	1.046	1.017	.524	1.064	1.035	.515	1.045	1.021
1	1	.516	1.048	1.019	.493	1.001	**.974**	.492	.998	.975
4/3	2	.469	**.937**	**.940**	.512	1.024	1.018	.464	**.928**	**.930**
5/3	2	.512	1.024	1.027	.499	.998	.992	.513	1.025	1.028
2	2	.510	1.020	1.023	.496	.992	.987	.504	1.007	1.010
Av. 2	Av. 0	.451	.907	.896	.500	1.007	.991	.452	.909	.900
Av. 1	Av. 0	.514	1.035	1.022	.512	1.031	1.014	.514	1.035	1.024
Av. 0	Av. 0	.513	1.034	1.021	.495	.997	.980	.498	1.002	.992
Using final-revised data										
0	1	.425	**.869**	**.820**	.471	**.963**	1.044	.440	**.900**	**.840**
1/3	1	.409	**.837**	**.790**	.472	**.966**	1.047	.438	**.897**	**.837**
2/3	1	.456	**.932**	**.880**	.474	**.969**	1.051	.484	.991	**.925**
1	1	.536	1.098	1.036	.477	**.975**	1.057	.521	1.067	.996
4/3	2	.459	**.965**	**.964**	.466	**.981**	.995	.469	**.987**	**.984**
5/3	2	.491	1.033	1.032	.472	.994	1.009	.495	1.043	1.039
2	2	.487	1.026	1.025	.479	1.008	1.023	.490	1.031	1.027
Av. 2	Av. 0	.435	.902	.874	.469	.973	1.021	.454	.942	.907
Av. 1	Av. 0	.473	.983	.952	.473	.981	1.029	.490	1.017	.979
Av. 0	Av. 0	.512	1.063	1.031	.478	.991	1.040	.506	1.049	1.010

NOTE: The entries (except the first column of each panel) are ratios of RMSE of MIDAS–AR to the AR and ADL. The RMSEs are computed using final-revised actual values of output growth for forecasts of 1985:Q2–2005:Q1. The ratios in bold imply that the null of equal RMSE is rejected at the 5% significance level using bootstrapped critical values. Av. 0 is the average RMSE when no information on the indicator in the current quarter is used for forecasting ($h = 1, 2$). Av. 1 is the equivalent when only 1 month on the current quarter is available ($h = 2/3, 5/3$). Av. 2 is the same measure when 2 months of information are available ($h = 1/3, 4/3$).

The exercise using final-revised vintage data allows us to calculate critical values to determine whether the gains that we observe from MIDAS at the shorter horizons are statistically significant. Following in the tradition of testing for equal predictive ability of West (1996) and West and McCracken (1998) (also see the review in West 2006), we employ the test of equal forecast accuracy for multistep forecasts from nested models of Clark and McCracken (2005). The aim is to compare the forecast performance in population. Note that the MIDAS–AR nests both the AR and ADL models; it specializes to the ADL when $\theta_1 = \theta_2 = 0$ in (2). The null is that the quarterly frequency AR (ADL) model forecasts are as accurate on RMSE as those of the MIDAS–AR (the unrestricted model), and the one-sided alternative is that the AR (ADL) model forecasts are less accurate. Because the test has a limiting distribution that depends on the data, we adopt a bootstrap implementation of the test (similar to that of Kilian 1999). The boldface entries in the table indicate that the null of equal RMSE is rejected at the 5% level using the bootstrapped critical values, so that the gains from using MIDAS at the shorter horizons clearly are significant for both industrial production and capacity utilization.

Testing for equal predictive ability is more complicated in real-time forecasting exercises when there are data revisions across the vintages. Clark and McCracken (2007) showed that data revisions may affect the asymptotic behavior of tests of equal forecast accuracy and suggested a way to proceed using linear models estimated by least squares. It is unclear how useful these results are in our context. The natural solution of using a bootstrap would require specification of the (unknown) revision process. For the forecasting comparisons based on the real-time use of end-of-sample and real-time vintage data, we simply report the relative sizes of the Root Mean Squared Forecast Errors (RMSFEs) and dispense with the use of formal tests of equal predictive ability.

Now consider the real-time end-of-sample vintage data. The results clearly indicate that IP and CU help predict output growth in real time when we have access to monthly data on the quarter being forecast. When only information on the previous quarter is used, the indicators are of no value in real time. These results essentially match those obtained using final-revised data. In real time, the gains to MIDAS relative to the AR disappear when EMP is the indicator, suggesting that the apparent gains from using EMP to predict output growth are not attainable in practice. In real time, the gains from using EMP at the quarterly frequency also disappear (i.e., comparison of the ADL against the AR). The boldface entries in the table indicate that the null of equal RMSE is rejected at the 5% level using the bootstrapped critical values calculated for the exercise using the final-revised data, but, as noted, the use of these critical values is questionable when there are data revisions. Nevertheless, it is apparent that MIDAS results in marked reductions in empirical RMSFEs in real time for the shorter horizons.

3.3 MIDAS–AR versus Other Methods of Exploiting Monthly Indicators

We compare the MIDAS–AR with other ways of using monthly indicator data, the MF–DL and the ADL–F described in Section 2.3, using end-of-sample vintage data. When all of the months of the quarter being forecast are known, ADL–F corresponds to a standard ADL, because data on the current quarter value of the indicator are available. Broadly speaking, the results given in Table 2 indicate that there is little to choose from between the models. The MF–DL and ADL–F are better than MIDAS for nowcasting ($h = 0$), but when only 1 or 2 months of indicator data are available, MIDAS is generally at least as good (see the averages of the ratios across different horizons). Overall, except when $h = 0$, the performance of MIDAS is promising compared with that of other simple methods of using monthly indicators to forecast quarterly growth. However, Koenig et al. (2003) showed that using real-time vintage data improves the forecast accuracy of MF–DL and advocated this approach to forecast accuracy comparisons. The next section reports a comparison using real-time vintage data.

3.4 MIDAS–AR and Other Methods of Exploiting Monthly Indicators With Real-Time Vintage Data

Conducting the forecasting exercise using real-time vintage data supports the finding that monthly indicator information helps predict output growth, especially at shorter horizons. This

Table 2. Comparing the forecasting performance of MIDAS–AR with ADL–F and MF–DL using real-time end-of-sample vintage data

	Industrial production (IP)		Employment (EMP)		Capacity utilization (CU)	
h	MF–DL	ADL–F	MF–DL	ADF–F	MF–DL	ADL–F
0	1.034	1.032	1.054	1.088	1.017	1.030
1/3	.981	.989	.965	1.040	.981	1.000
2/3	.999	1.013	.978	.985	.995	1.023
1	.993	1.019	.957	.974	.995	.975
4/3	1.010	.974	.966	1.015	1.009	.978
5/3	1.002	.947	.979	.939	.996	.958
2	1.008	1.023	.999	.987	.979	1.010
Av. 2	.996	.981	.965	1.027	.996	.988
Av. 1	1.000	.979	.979	.962	.995	.989
Av. 0	1.000	1.021	.977	.980	.987	.992

NOTE: The entries are ratios of the MIDAS–AR RMSEs to the RMSEs of the stated models. Av. 0 is the average RMSE when no information on the indicator in the current quarter is used for forecasting ($h = 1, 2$). Av. 1 is the equivalent when only 1 month on the current quarter is available ($h = 2/3, 5/3$). Av. 2 is the same measure when 2 months of information are available ($h = 1/3, 4/3$).

Table 3. Comparing the forecasting performance of MIDAS–AR with that of MF–DL, ADL–F, and AR using real-time vintage data and end-of-sample vintage data

	Industrial production (IP)			Employment (EMP)			Capacity utilization (CU)			
h	MIDAS–AR	MF–DL	ADL–F	MIDAS–AR	MF–DL	ADL–F	MIDAS–AR	MF–DL	ADL–F	AR
Real-time vintage data/end-of-sample vintage data										
0	.971	1.005	.990	.961	1.034	.952	.996	1.041	.960	
1/3	.974	.971	.962	.973	.988	.970	1.011	1.034	.971	
2/3	.993	.988	1.124	.959	.968	.937	1.027	1.029	1.066	
1	1.018	1.048	.984	1.036	1.029	1.008	1.098	1.105	.991	1.035
4/3	1.028	1.066	1.000	1.001	1.001	.995	1.019	1.023	.963	
5/3	.996	1.024	1.071	1.015	1.018	.965	1.014	1.042	1.069	
2	1.029	1.050	1.014	1.029	1.065	1.029	1.020	1.021	.991	1.012
Ratio of MIDAS–AR with real-time vintage data to										
	AR	MF–DL	ADL–F	AR	MF–DL	ADL–F	AR	MF–DL	ADL–F	
Av. 2	.890	.978	1.001	.972	.958	1.031	.902	.983	.995	
Av. 1	1.006	.989	.888	.993	.973	.998	1.033	.981	.961	
Av. 0	1.034	.976	1.046	1.006	.965	.994	1.038	.984	1.029	

NOTE: The entries in the top part are RMSE ratios of the stated model estimated with real-time vintage data to the same model but estimated with end-of-sample vintage data. The entries in the bottom part are ratios of the MIDAS–AR RMSEs to the RMSEs of the stated models using real-time vintage data. Av. 0 is the average RMSE when no information on the indicator in the current quarter is used for forecasting ($h = 1, 2$). Av. 1 is the equivalent when only 1 month on the current quarter is available ($h = 2/3, 5/3$). Av. 2 is the same measure when 2 months of information are available ($h = 1/3, 4/3$).

is partly because the forecast performance of the AR worsens relative to when end-of-sample data are used (by just over 3% on RMSE at $h = 1$), whereas the performance of MIDAS and ADL–F generally improves. Table 3 gives the ratios of the RMSEs when using real-time vintage data to using end-of-sample vintage data for each monthly model and the AR. The net result of these changes is that MIDAS is more accurate than the AR for $h = 0$, $1/3$ for all three indicators. The table also shows that for horizons with 2 months of indicator information available (the "Av. 2" row), the gains to MIDAS are of the order of a 10% reduction in RMSE for IP and CU and a 3% reduction for EMP.

It is worth remarking that MIDAS is generally as good at the shorter horizons as the popular two-step approach to the incorporation of monthly information exemplified by the ADL–F, except when the indicator is EMP, and is almost always superior to the MF–DL.

3.5 Forecast Combination

Table 4 compares models that include all three indicators (which have the prefix "M–," for multiple) with equal-weighted combinations of the individual indicator models. The results indicate that the MIDAS model is clearly preferred to MF–DL. It also beats ADL–F when the horizon is not an integer multiple of quarters, that is, for those horizons when ADL–F relies on monthly forecasts to construct quarterly observations. Combining forecasts is better than combining indicators within a single model (as is often the case; see, e.g., Clements and Galvão 2006), but the combinations of the MIDAS models' forecasts are generally at least as good as the combinations of the forecasts from the other two models.

4. CONCLUSIONS

In recent years, increasing use has been made of monthly indicator information to generate forecasts of quarterly macro aggregates, such as GDP growth. We have investigated whether the MIDAS approach of Ghysels et al. (2004, 2006b) can be successfully adapted to short-term forecasts of output growth, given that it has hitherto been used for forecasting financial variables with daily observations. A typical feature of quarterly macroeconomic time series is that they often can be reasonably well modeled by AR processes. To capture this characteristic of macro data, we extended the distributed-lag MIDAS specification to include an AR term (the MIDAS–AR) and showed how this model can be applied in a forecasting context.

Recent research suggests that the predictive content of indicator information must be assessed in an exercise that mirrors a "real-time-forecasting environment," and that using final-revised data may misleadingly suggest that the indicators are better than what could be achieved with the data available at the time. We conducted a traditional real-time forecasting exercise that exploits the monthly vintages of the indicators and the quarterly vintages of output growth and that is consistent with the timing of the releases of the different data vintages. This permits a comprehensive and valid appraisal of the usefulness of monthly information in real time. We found that the use of monthly indicator information resulted in sizeable reductions in RMSE for short-horizon forecasts when within-quarter monthly data on industrial production and capacity utilization are used.

We also evaluated the suggestion by Koenig et al. (2003) to base the real-time forecasting exercise on real-time vintage data as opposed to end-of-sample vintage data. We did so for a range of forecast horizons, from the "nowcasts" considered by Koenig et al. up to the 2-quarter horizon, in steps of 1 month. The use of real-time vintage data serves to strengthen our finding that within-quarter monthly indicator information can result in marked improvements in forecast performance. MIDAS fares well relative to the other models that use monthly information. Coupled with its flexibility and ease of use relative to methods

Table 4. Comparing combinations of indicators: M–MIDAS–AR, M–ADL–F, M–MF–DL, and means of forecasts using real-time vintage data

	Multiple indicator models			Combining forecasts		
h	M–MIDAS–AR	M–MF–DL	M–ADL–F	Mean MIDAS–AR	Mean MF–DL	Mean ADL–F
0	.442	.450	.432	.420	.426	.420
1/3	.425	.441	.431	.412	.421	.417
2/3	.520	.522	.543	.490	.487	.507
1	.547	.565	.511	.513	.519	.500
4/3	.498	.516	.486	.478	.481	.478
5/3	.519	.573	.594	.510	.518	.537
2	.526	.563	.513	.514	.522	.509
Av. 2	.463	.480	.459	.446	.452	.448
Av. 1	.519	.548	.569	.500	.502	.522
Av. 0	.536	.564	.512	.514	.520	.504
	Ratio of M–MIDAS–AR to			Ratio of mean MIDAS–AR to		
	M–MF–DL	M–ADL–F	AR	Mean MF–DL	Mean ADL–F	AR
0	.983	1.023	.898	.984	.999	.852
1/3	.965	.987	.863	.979	.989	.837
2/3	.996	.957	1.054	1.007	.966	.995
1	.967	1.071	1.109	.989	1.027	1.042
4/3	.964	1.024	.995	.993	1.000	.954
5/3	.906	.874	1.037	.985	.950	1.019
2	.934	1.026	1.051	.986	1.010	1.028
Av. 2	.965	1.008	.932	.987	.995	.898
Av. 1	.948	.913	1.046	.996	.957	1.007
Av. 0	.951	1.048	1.080	.987	1.019	1.035

NOTE: The entries in the top part are RMSEs. The bottom part records the RMSE ratios of MIDAS to the stated model. Av. 0 is the average RMSE when no information on the indicator in the current quarter is used for forecasting ($h = 1, 2$). Av. 1 is the equivalent when only 1 month of information for the current quarter is available ($h = 2/3, 5/3$). Av. 2 is the same measure when 2 months of information are available ($h = 1/3, 4/3$).

that involve generation of forecasts of explanatory variables offline, the MIDAS–AR would appear to be a useful addition to the sets of models and methods that exploit monthly indicators for the short-term forecasting of macroaggregates.

ACKNOWLEDGMENTS

The authors are grateful to the editor, an associate editor, and three anonymous referees for helpful comments and guidance. They also acknowledge helpful comments from participants at the International Symposium of Forecasting in Santander, June 2006.

[Received September 2005. Revised November 2007.]

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