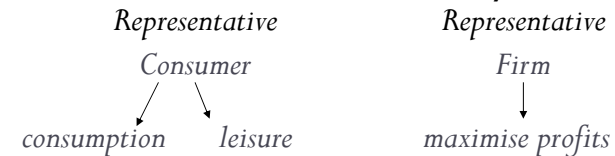


## The Static Model

Dr. Ana Beatriz Galvao; Business Cycles;  
Lecture 2; Queen Mary; 2012

## A description of the Model Economy

► **Static:** decisions are made for only one time period.



- **Rational agents** (consumers and firms) that are able to make an optimising decision.
- Agents take decisions in a **competitive market**: market prices are outside their control.
- A **barter economy**: no monetary changes, so goods are exchanged for goods.
- **Government** collects lump-sum taxes (so tax decisions do not depend on the decision of agents).

► 2

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## Consumer

- The preferences of the consumer over leisure and consumption goods are described by the utility function:

$$U(C, l)$$

- Even if we cannot measure the utility, we know the preference of the consumer between two bundles:

$$U(C_1, l_1) > U(C_2, l_2)$$

<

=

indifferent

► 3

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## Consumer

Assumptions on the **preferences**:

1. More is always preferred to less (nonsatiation, monotonicity).
2. The consumer likes diversity in his/her consumption bundle (convexity).
3. Consumption and leisure are normal goods (purchases more when income increases).

**Indifference curve:** the set of all consumption bundles (combinations of C and l) that the consumer is indifferent.

Properties of Indifference curve (consequence of assumptions on preferences):

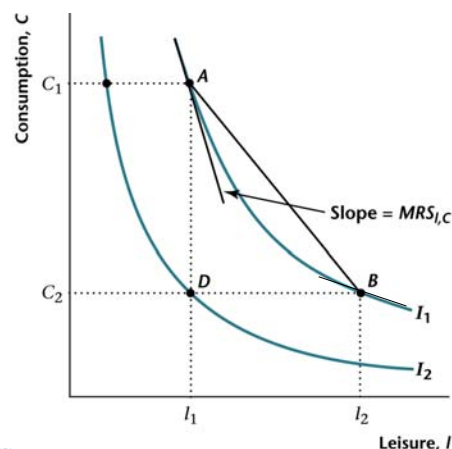
- (A) Curve is downward sloping.
- (B) Curve is convex.

► 4

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## Consumer

- ▶ **Marginal Rate of Substitution:** rate that the consumer is willing to substitute leisure for consumption goods.



$MRS_{l,C} = -[\text{slope of indifference curve}] = -[\Delta C / \Delta l]$

How much one is willing to give up of consumption to consume more leisure.

Because the absolute value of the slope decreases with  $l$ , this means that MRS decreases with  $l$ .

## Consumer: Budget Constraint

Time constraint:

$$l + N^S = h$$

Hours spent working;  
Labour supply.

Total hours available

- ▶ Wage ( $w$ ) is expressed in terms of the consumer good.
- ▶ The real wage income is  $wN^S$ .
- ▶ The consumer also receives a dividend income ( $\pi$ ) from the firms.
- ▶ He/she pays a lump-sum tax ( $T$ ) for the government.

6

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## Consumer: Budget Constraint

The consumer budget constraint is:

$$C = \underbrace{wN^S + \pi - T}_{\text{Total disposable income.}}$$

The total market expenditure is equal to the disposable income with substitution of time constraint:

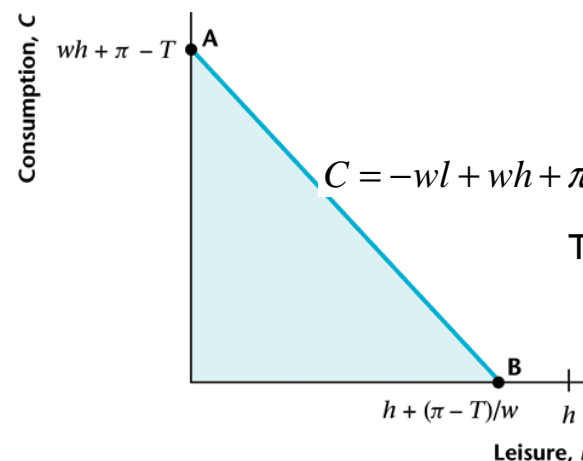
$$C = w(h - l) + \pi - T$$

Other interpretation:

$$C + \underbrace{wl}_{\text{Economic value of time}} = wh + \underbrace{\pi - T}_{\text{Disposable income from dividend}}$$

Cost of consumption of leisure

## Consumer: Budget Constraint



$T > \pi$  (consumer needs to work to pay taxes)

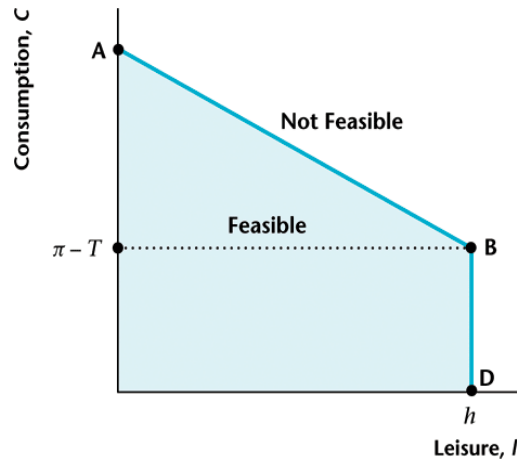
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7

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## Consumer: Budget Constraint



$\pi > T$ , so the consumer may purchase consumption goods without working if it is in the region between B-D

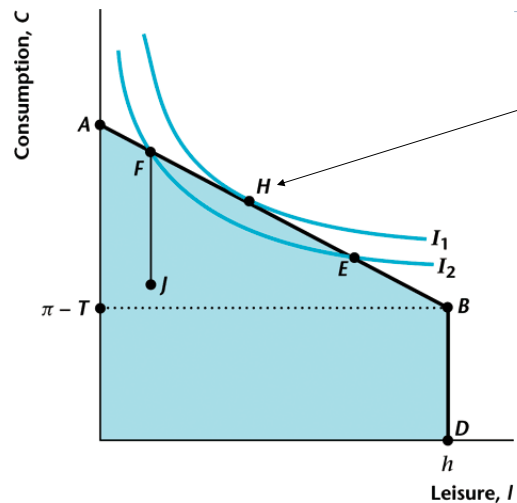
## Consumer: Optimisation

- “The optimal consumption bundle is the point representing a consumption-leisure pair that is on the highest possible indifference curve and is on or inside the consumer’s budget constraint”.

$$\max_{C,l} U(C,l)$$

Subject to:  $C = w(h - l) + \pi - T$

## Consumer: Optimisation

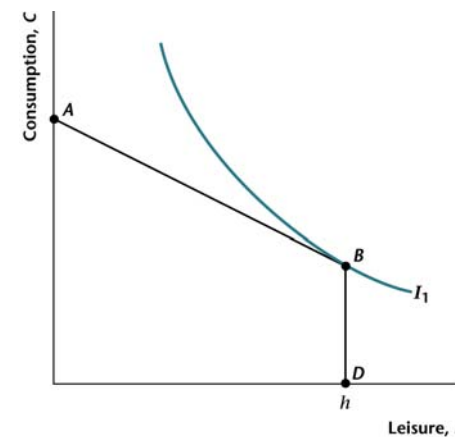


$$MRS_{l,C} = w$$

Relative price of leisure in terms of consumption goods.

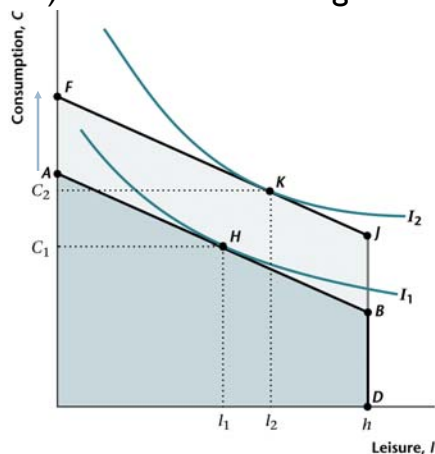
## Consumer: Optimisation

- ▶ The consumer may choose not work:



## Consumer: Increase in $\pi$ -T

- ▶ Consumption and leisure increase (they are both normal goods). Values in K are larger than in H



▶ 13

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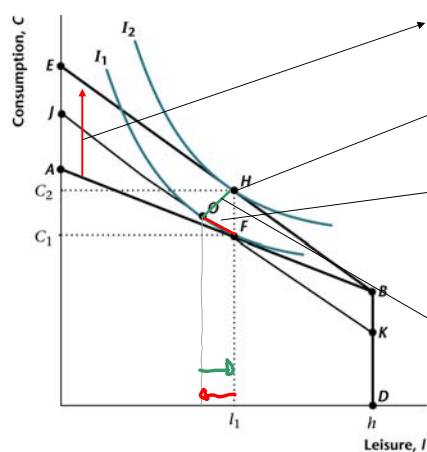
## Consumer: increases in Real wage

- ▶ An increase in real wage has an **income** and a **substitution effect**.
- ▶ The **income effect** increases both consumption and leisure because they are both normal goods.
- ▶ The **substitution effect** implies that leisure falls. This arises from the fact that the cost of leisure increases. The substitution effect is captured without changing the indifference curve but using the slope of the budget constraint after increasing the real wage.
- ▶ The total effect of increase in real wage on leisure is ambiguous.

▶ 14

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## Consumer: increases in Real wage



The slope of the budget constraint increases.

New optimal choice with higher indifference curve.

**Substitution effect:** same indiff. Curve, but budget const. with the new real wage. Work more hours!

**Income effect:** same real wage, but higher indifference curve. Increases consumption and leisure.

▶ 15

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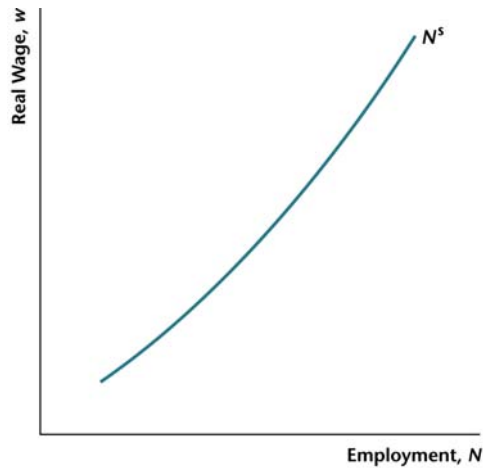
## Labour Supply

- ▶ Labour supply is how much labour the representative consumer wishes to supply given a real wage.
  - ▶ If the **substitution effect is larger than the income effect** when real wage increases, the labour supply has a positive slope.
- $$N^S(w) = h - l(w)$$
- ▶ But for larger values of  $w$ , one can find income effect larger than substitution effect (backward-bending labour supply).

▶ 16

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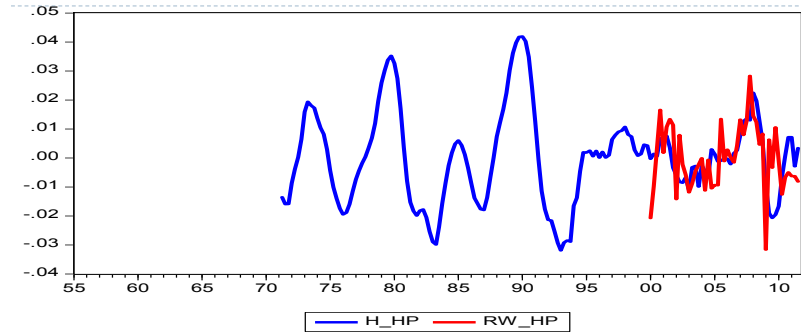
## Labour Supply



▶ 17

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## Real Wage X Hours in the UK



Hours are increasing with real wage: substitution effect!  
(Correlation is .4).

▶ 18

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## Firm

- ▶ Production Function (technology) of the representative firm:

$$Y = zF(K, N^d)$$

↙
↖
↖

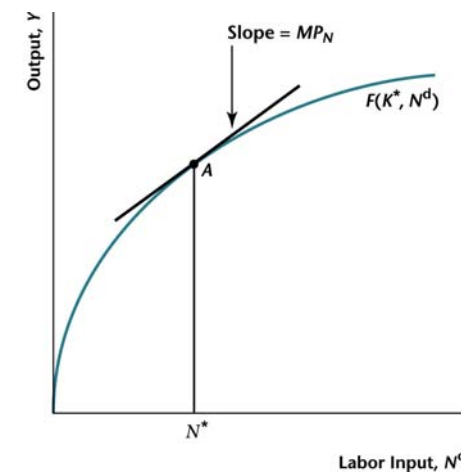
Total factor productivity:
capital
labour

degree of sophistication of
the production process.

▶ 19

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## Firm

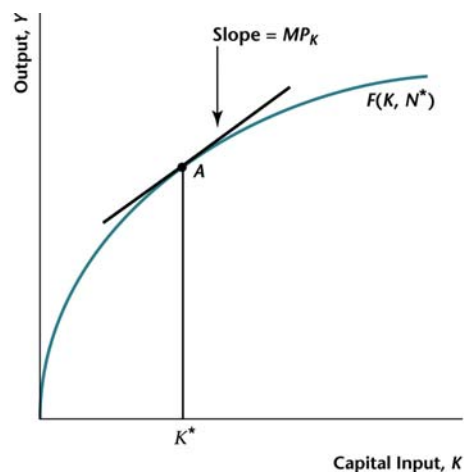


Marginal product of labour is the additional output that can be produced with one additional unit of labour input, holding constant the capital.

▶ 20

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## Firm

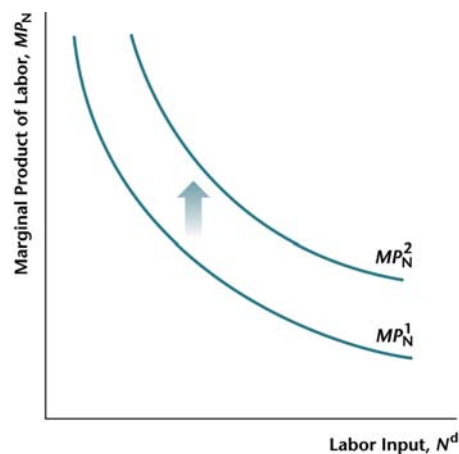


Marginal product of capital is the additional output that can be produced with one additional unit of capital input, holding constant the labour.

## Firm

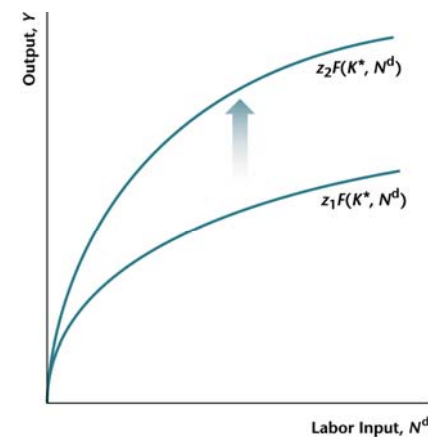
- ▶ Properties of the Production Function:
- ▶ (1) The production function of the representative firm has **constant returns to scale**. This implies that  $zF(cK, cN^d) = czF(K, N^d)$ .
- ▶ (2) The marginal product of capital and labour are positive and they are decreasing with the quantities (**diminishing marginal product**).
- ▶ (3) The marginal product of labour increases as the quantity of capital input increases.

## Firm



Effect of increasing capital: the labour productivity increases (marginal product).

## Firm



Effect of increasing total factor productivity: production function moves up and its slope increases.

## Firm: Profits

$$\pi = zF(K, N^d) - wN^d$$

Total revenue that firms receives from selling its output.

Total real cost of labour (wage is given).

## Firms: Maximisation

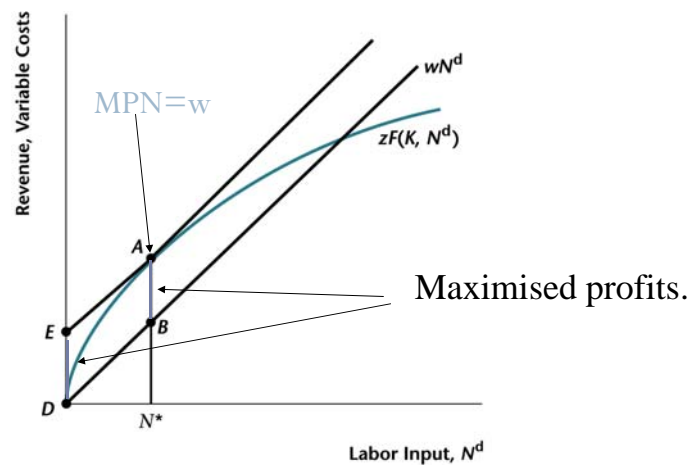
$$\max_{N^d} (zF(K, N^d) - wN^d)$$

FOC:

$$zF_2(K, N^d) = w$$

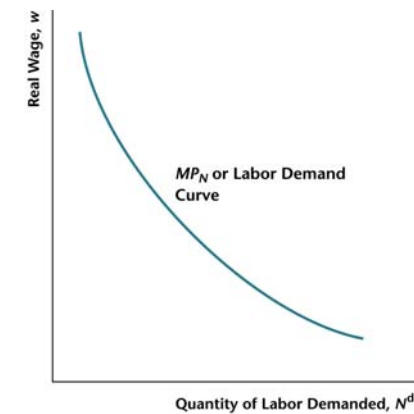
Marginal product of labour, depends on capital and total factor productivity.

## Firms: Maximisation



## Labour Demand

- ▶ Because  $MP_N = w$  when the firm maximises profit, then the demand for labour is the marginal product of labour.



## The Government

- ▶ The government finances the purchase of a given quantity of consumption goods  $G$  by taxing the representative consumer (lump-sum tax).
- ▶ Because the government cannot borrow, it has the following budget constraint:

$$G = T$$

- ▶  $G$  is exogenous while  $T$  is endogenous.

## The Model Economy

### Exogenous variables

Government  
Expenditure:  $G$   
Capital Stock:  $K$   
Total Factor  
Productivity:  $z$

### Endogenous variables

Consumption:  $C$   
Labour supply:  $N^s$   
Labour demand:  $N^d$   
Lump-sum taxes:  $T$   
Aggregate Output:  $Y$   
Real Wage:  $w$

## Competitive Equilibrium

- ▶ All consumers and firms are price-takers, so the economy is in equilibrium when demand is equal to supply in all markets, that is, the **markets clear**.
- ▶ There is only one market in this economy (labour market) and one price (real wage), so the equilibrium is achieved when, given the exogenous variables, the real wage is such that the quantity of labour the consumer wishes to supply is equal to the quantity of labour the firm wishes to hire.
- ▶  $T$  must satisfy government budget constraint and dividends ( $\pi$ ) must be equal to profits.

## Competitive Equilibrium

- ▶ The following conditions are satisfied in the competitive equilibrium:
  1. Consumer chooses  $C$  and  $N^s$  subject to budget constraint ( $w, T, \pi$  are given).
  2. Firm chooses  $N^d$  in order to maximise profits given  $z, K$  and  $w$ .
  3. The market for labour clears, that is,  $N^s = N^d$
  4. The government budget constraint is satisfied ( $G = T$ ).

## Implication of the Comp. Equilibrium

- ▶ If the firms maximise profits and the government budget constraint holds, the budget constraint of the consumer is:  $C = wN^S + \pi - T$

$$C = wN^S + (y - wN^D) - G$$

In equilibrium,  $N^S = N^D = N$

$$C = y - G$$

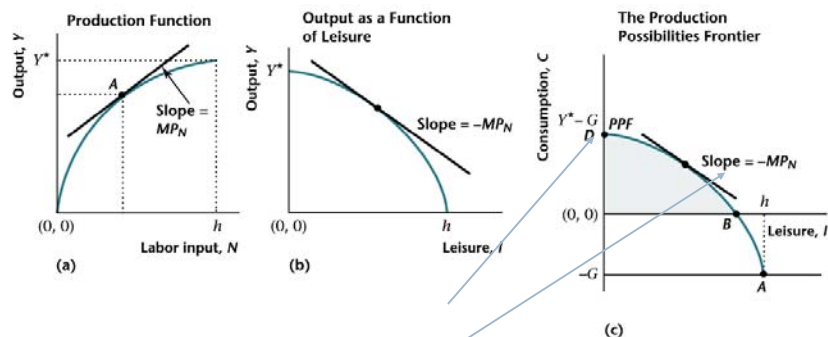
$$y = C + G$$

Traditional equation of macroeconomics!

## The Production Possibilities Frontier

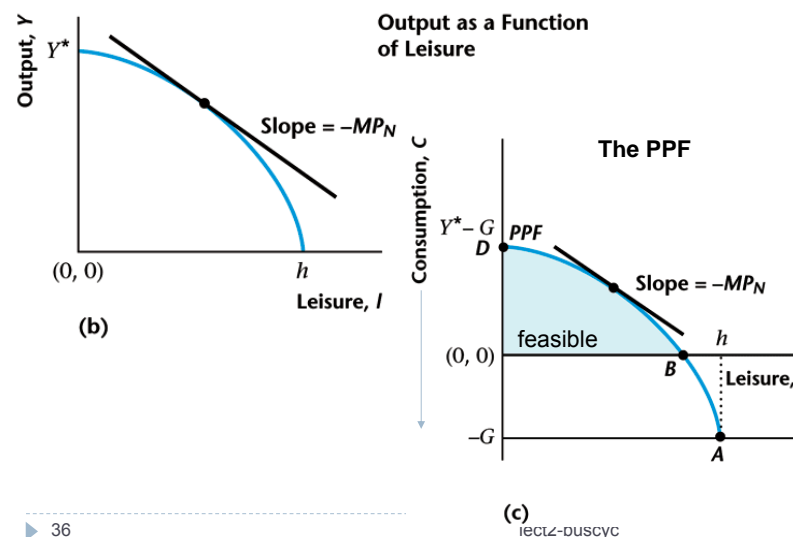
- ▶ The boundary of a set that describes what consumption bundles (consumption good and leisure) are technologically feasible to produce.
- ▶ In equilibrium, we can re-write the technology of the firm as:  $Y = zF(K, N)$   
 $Y = zF(K, h-l)$
- ▶ And because  $C=Y-G$ , the possible combinations of consumption goods and leisure is:  
 $C = zF(K, h-l) - G$

## The Production Possibilities Frontier

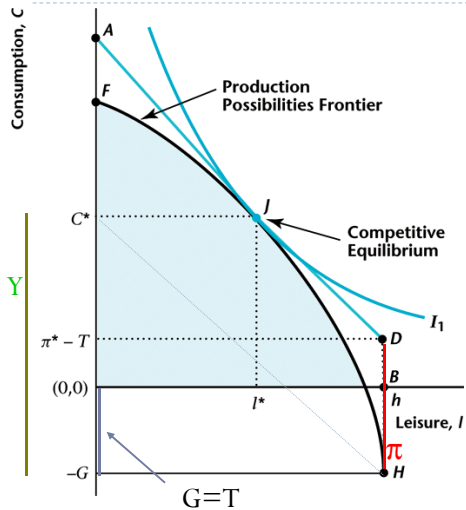


Marginal Rate of Transformation is the rate at which leisure can be converted technologically to the consumer good. It is equal to the marginal product of labour.

## The Production Possibilities Frontier



## Competitive Equilibrium

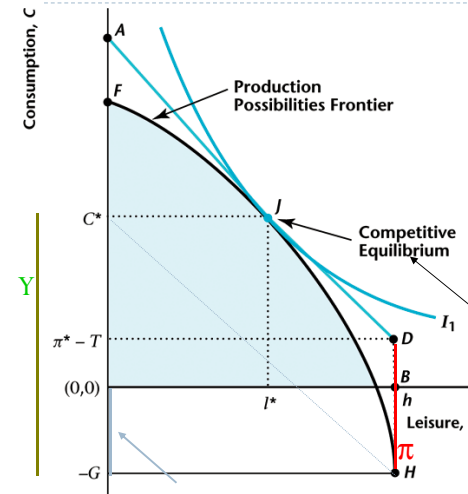


1. AD line has  $w$  as slope, so at J:  $MP_N = w$  and  $MRT = MP_N = w$ .
2. Point J maximises profits because  $MP_N = w$
3. The Government Budget constraint is satisfied because  $G = T$  at J.

▶ 37

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## Competitive Equilibrium



1. ADB is the consumer budget constraint.
2. Point J maximises utility because  $MRS = w$ .

$$MRS = MRT = MP_N = w$$

▶ 38

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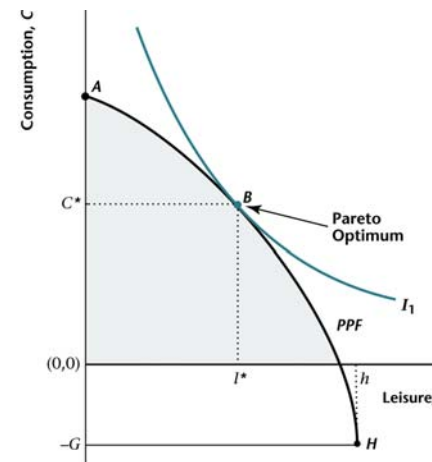
## Pareto Optimal

- ▶ A competitive equilibrium is Pareto optimal if there is no way of rearrange production or to reallocate goods so that someone is made better off without making someone else worse off.
- ▶ The competitive equilibrium is Pareto optimal if has the same combination of C and I that a benevolent social planner would choose.
- ▶ The benevolent social planner can coerce the firm, the consumer and the government to choose the most efficient combination.

▶ 39

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## Pareto Optimal



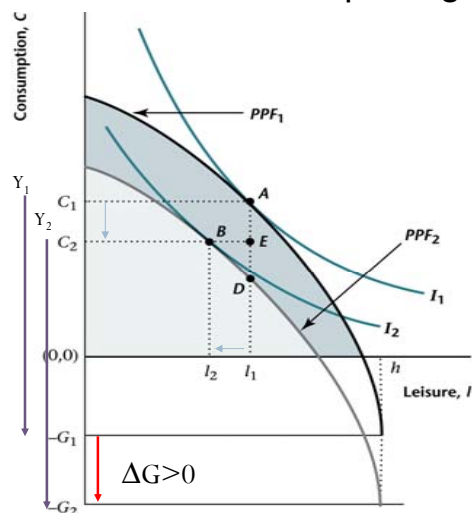
- ▶ Pareto optimal choice is the one that the highest indifference curve intercepts the production possibilities frontier.
- ▶ It is the same bundle as the competitive equilibrium because at B,  $MRS = MRT = MP_N$ .

▶ 40

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## Increases in Government Purchases

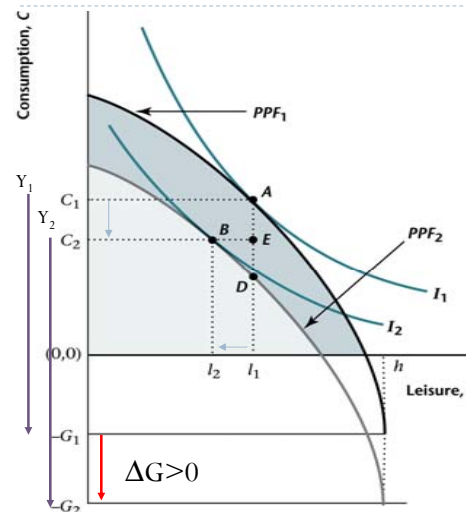
▶ Does Government spending cause business cycles?



$$C = zF(K, h-l) - G$$

- $\Delta G > 0$  is represented by a downward shift of the production frontier.
- Consumers purchase less goods and work more.
- Firms produce more.
- AE is the crowding out effect.

## Increases in Government Purchases



- Because consumers work more,  $MP_N$  at B is smaller than at A (diminishing returns).
- New equilibrium has smaller marginal product of labour (transformation), so the real wage decreases ( $w_2 < w_1$ ).
- Indifference curve is lower, so  $MRS = MP_N$  decreases.

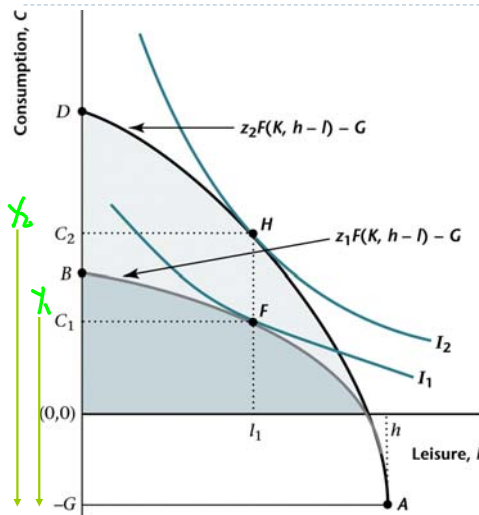
## Increases in Government Spending

- $\Delta G > 0 \rightarrow$  aggregate output and employment increase; consumption and the real wage decrease.
- Business cycle features: consumption and real wages are **countercyclical**, so it is unlikely that government spending would be the cause of business cycles based on this model economy.

## Increases in Total Factor Productivity

- ▶ Technological changes.
- ▶ The production function moves upward and the inclination increases (the marginal product of labour increases).
- ▶ The production possibilities frontier (PPF) shifts upward, so more consumption is obtainable for a given quantity of leisure. The PPF is also steeper.

## Increases in Total Factor Productivity

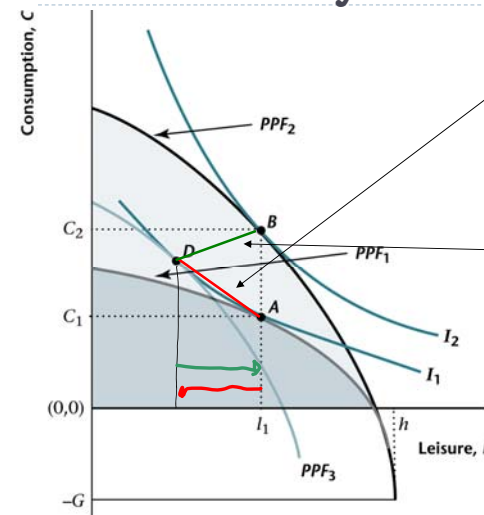


- ▶ Aggregate output increases with new technology.
- ▶ Consumption increases because consumers may consume more for the same quantity of leisure.
- ▶ Leisure (employment) may increase or not.
- ▶ Government Spending does not change.

▶ 45

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## Increases in Total Factor Productivity



- ▶ **Substitution effect:** same indifference curve, but MRT of  $PPF_3$  equals to  $PPF_2$  (after increase in TFP). Effect on leisure is negative.
- ▶ **Income effect:** same MRT of  $PPF_2$ , but higher indifference curve. Effect on leisure is positive
- ▶ Real wage in B is larger than A because the MRS is larger at D than at A (and D is parallel to B). Recall that  $MRS = MP_N$  at B.

▶ 46

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## Increases in Total Factor Productivity

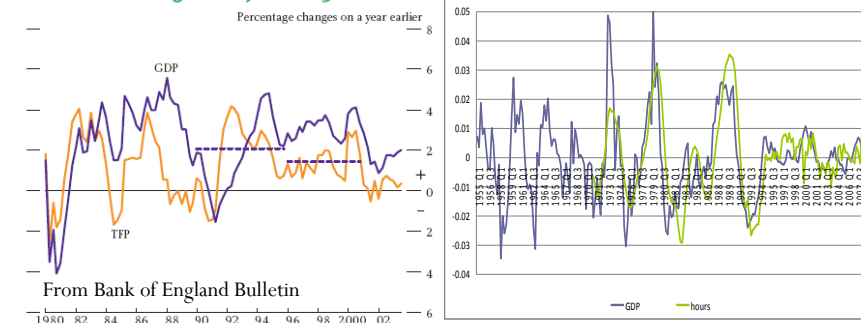
- $\Delta z > 0 \rightarrow$  aggregate output, consumption and real wage increase, but the effect on hours of work (hours per work and employment) is uncertain.
- However, if the substitution effect is larger than the income effect, then employment is procyclical because increases when output increases.
- Therefore, TFP shocks + static model are able to deliver the business cycles stylised facts.

▶ 47

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## TFP and the UK Cycle

Chart 1  
Growth of total factor productivity and GDP for the United Kingdom: 1980–2003



A positive technological shock will make workers to choose less leisure because they are more productive, so the cost of an hour of leisure increases.

▶ 48

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