Changes in Predictive Ability with Mixed Frequency Data

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Abstract

This paper proposes a model with regime changes in the ability of a high frequency variable in predicting a low frequency variable, called smooth transition mixed data sampling (STMIDAS) regression. Simulation exercises indicate that improvements in forecasting accuracy from the use of mixed data sampling are more sizeable in nonlinear than in linear specifications. Real-time out-of-sample results suggest that recurrent changes in predictive ability are more important to improve forecasts of output growth using financial variables than the direct use of financial data sampled weekly/daily.

Key words: smooth transition, MIDAS, predictive ability, financial indicators, economic activity

JEL codes: C22, C53, E44

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1 Introduction

Asset prices are forward-looking and incorporate expectations about future economic activity; as a result, bond and stock returns should be useful predictors of economic activity (Harvey, 1988; Stock and Watson, 2003). Empirical out-of-sample evaluations have found instabilities on the predictive power of asset prices for economic activity (see, Stock and Watson (2003) for a survey). There is also evidence of asymmetries when the spread between long and short-term interest rates is employed to predict output growth (Galbraith and Tkacz, 2000; Anderson and Vahid, 2001; Galvão, 2006). Finally, the direct use of financial indicators sampled daily instead of aggregating observed data to the quarterly frequency may improve the accuracy of economic activity forecasts (Ghysels and Wright, 2009; Andreou, Ghysels and Kourtellos, 2009).

This paper proposes a model in which the predictor is sampled at a frequency higher than the target variable, and the predictor's impact on the future value of the target variable changes with regime. I use the new specification for evaluating the relative forecasting contributions of nonlinearities and of the direct use of daily/weekly data. Recurrent regime changes depend on deviations of the weighted high frequency predictor with respect to a threshold within a smooth transition function. This specification design leads to asymmetries in the dynamic relation between the predictor and the target variable.

Specifically, the model allows for mixed data sampling (Ghysels, Santa-Clara and Valkanov, 2004) in smooth transition regressions (Teräsvirta, 1998). The estimation of aggregation weights enhances the efficiency on the estimation of transition function parameters of a typical smooth transition regression. Simulation results indicate that we should expect more sizeable improvements in forecasting accuracy from the use of mixed data sampling in nonlinear than in linear specifications. These new simulation results based on a nonlinear regression complement the assessment of Andreou, Ghysels and Kourtellos (2010) about consequences of mistaken aggregation weights in linear regressions.

Previous forecasting evaluations[1] support the claim that the direct use of monthly and daily

[1] See Clements and Galvão (2008), Clements and Galvao (2009) and Kuzin, Marcellino and Schumacher (2009) for forecasts computed with monthly indicators, and Ghysels and Wright (2009) and Andreou et al. (2009) for forecasts

data improves nowcasts – current quarter forecasts – of output growth. The main interest of this paper is on the relative contributions of estimating aggregation weights and of modelling asymmetric predictive power. Using a MIDAS regression, it is simple to use daily financial data already observed in the current quarter to predict a not yet published economic activity variable measured in the current quarter. However, when computing forecasts from a linear regression with fully aggregate data, all daily observations up to the end of current quarter are required before computing nowcasts. This informational advantage given to MIDAS regressions by the use of leads (as considered by Clements and Galvão (2008) and Andreou et al. (2009)) is not exploited in this paper; as a result, only typical forecast horizons are considered to evaluate the effect of both nonlinearities and direct use of high frequency data.

The literature suggests that improvements in forecasting accuracy from modelling nonlinearities are generally limited (Galvão, 2006; Anderson, Athanasopoulos and Vahid, 2007). For example, when computing forecasts of economic activity with the spread between long and short-term interest rates, the spread has predictive power for low and negative growth, but it is not a good predictor of periods of high growth (Galbraith and Tkacz, 2000). If the out-of-sample period does not include periods of low/negative growth, the nonlinear model may be at a disadvantage, and evidence of the predictive content of the spread may be hard to find.[2] Giacomini and Rossi (2006) provide evidence on the fact that the predictive content of spread for output growth has decreased over time based on a linear forecasting model. It may not be a coincidence that the period of low predictive power of the spread is also a period of strong economic growth. Therefore, the use of specifications that allow for those asymmetries could change our inference on the predictive ability of financial variables for economic activity.

The inference on the predictive ability is implemented using the fluctuation test of Giacomini and Rossi (2010) in order to take instability into account. This paper presents strong evidence of

from daily financial indicators.

[2] Anderson et al. (2007) support similar argument when forecasting economic activity using the spread. In addition, Terasvirta (2006) brings attention to short out-of-sample periods, which are likely to exclude periods that asymmetries are important, as a general problem of applying nonlinear models for forecasting.

instability on the predictive content of financial variables for forecasting output growth, in agreement with Stock and Watson (2003), Estrella, Rodrigues and Schich (2003), and Giacomini and Rossi (2006). In addition, I find evidence that the inclusion of nonlinearities may affect the inference on predictive ability.

In contrast with Andreou et al. (2009), I focus on a small subset of financial indicators with time series data available for a long period. Results of a monte carlo exercise, which will be described in section 2, suggest that reasonable large sample sizes are required (at least 100 quarterly observations) in order to observe significant forecast accuracy improvements from using nonlinear specifications. Empirical results on the use of financial variables to predict output growth in the literature are generally based on US data (Stock and Watson (2003) and Anderson et al. (2007) are exceptions). I also include out-of-sample forecast analysis of UK output growth, even though the sample size available is shorter. Data availability also explains computing forecasts only for the US and UK: real-time datasets of output growth are easily available for these two countries, including data vintages since the 70's. The availability of real-time data is an important issue because the inference on the predictive ability of indicators to forecast output growth may depend on using the time series actually available to a practitioner at each forecast origin (Diebold and Rudebusch, 1991; Faust, Rogers and Wright, 2003; Orphanides and Van Norden, 2005).

An out-of-sample comparison of US and UK output growth forecasts is presented in section 3, and it includes indicators as the spread, the short-term rate, and stock returns. The spread between long-term and short-term interest rates is one of the most popular leading indicators of US growth (Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002). Stock returns, in contrast, have only marginal content for predicting output growth as concluded by Stock and Watson (2003), although the results of Estrella and Mishkin (1998) suggest some power in predicting recessions at short horizons. Short-term interest rates are not as popular indicators as the spread, but recently, Ang, Piazzesi and Wei (2006) argue that short-rates are a better leading indicator than the spread from 1990 onwards.

2 Smooth Transition Mixed Data Sampling Regression

2.1 MIDAS Approach

Ghysels et al. (2004) proposed the MIDAS approach, which is aimed at using different sampling frequencies in a regression, so that a low frequency variable can be directly regressed on a high frequency variable. The MIDAS approach has been successfully applied to forecast quarterly macroeconomic series using monthly data (Clements and Galvão, 2008; Clements and Galvao, 2009; Kuzin et al., 2009) and daily data (Ghysels and Wright, 2009; Andreou et al., 2009).

A MIDAS regression that employs x_t for directly forecast y_t at h-steps ahead is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} w(L^{1/m}) x_t^{(m)} + \varepsilon_{t+h} \tag{1}$$

where $w(L^{1/m}) = \sum_{j=1}^{K} w(j) L^{j/m}$ is a polynomial in the lag operator $L^{1/m}$ such that $L^{j/m} x_t = x_{t-j/m}$. $\beta_{1,h}^{(m)}$ is the impact of one unit change in $x_t^{(m)}$on y_t at h-steps-ahead after x, which is sample at a frequency m times higher than y, is aggregated using weights $w(j)$. $\beta_{1,h}^{(m)}$ is identified if $\sum_{j=1}^{K} w(j) = 1$.

In the case that y_t is sampled quarterly and $x_t^{(m)}$ is sampled weekly, assuming that only current quarter information on x is used to predict y ($K = m = 13$), the MIDAS regression is:

$$y_{t+h} = \beta_{0,h}^{(13)} + \beta_{1,h}^{(13)} \left[w(1) x_t^{(13)} + w(2) x_{t-1/13}^{(13)} + \cdots + w(13) x_{t-12/13}^{(13)} \right] + \varepsilon_{t+h}.$$ [3]

A problem with this specification is that the number of parameters in $w(L^{1/m})$ increases with the frequency of the predictor. A solution is the use of a function to approximate the weights. A weighting function that depends on the vector of parameters $\kappa = (\kappa_1, \kappa_2, ..., \kappa_q)$ is:

$$w(j; \kappa) = \frac{f(j, \kappa)}{\sum_{j=1}^{K} f(j, \kappa)}.$$

Two specifications for $f(j, \kappa)$ are popular in the literature, Ghysels, Santa-Clara and Valkanov (2005), Clements and Galvão (2008), Kuzin et al. (2009), Andreou et al. (2009) use exponential polynomial

[3] Of course, not all quarters have 13 full weeks consequently, $m = 13$ is an approximation. Empirically, the last 13 weekly observations backwards from the date at the end of quarter are employed (assuming $K = m$). Similar reasoning applies when using daily data ($m = 62$).

functions:

$$f(j,\kappa) = \exp(\kappa_1 j + \kappa_2 j^2 + ... + \kappa_q j^q).$$

And Ghysels, Santa-Clara, Sinko and Valkanov (2007) use a beta function:

$$f(j,\kappa) = \frac{(k)^{\kappa_1 - 1}(1-k)^{\kappa_2 - 1}\Gamma(\kappa_1 + \kappa_2)}{\Gamma(\kappa_1)\Gamma(\kappa_2)}; k = j/(K+1) \tag{2}$$

Ghysels et al. (2007) argue that even with only two parameters, the beta function is flexible enough to accommodate different weighting shapes. For comparison purposes, the exponential function is also employed with two parameters in the remainder of this paper, that is,

$$f(j,\kappa) = \exp(\kappa_1 j + \kappa_2 j^2). \tag{3}$$

The main advantage of MIDAS regressions is to give an opportunity to consider information on x_t to forecasting y_{t+h} that may otherwise be smoothed out after aggregation. It also improves efficiency of estimates of intercept and slope parameters, and may eliminate a bias created by aggregation when the predictor is described by an autoregressive process (Andreou et al., 2010).

Both weighting functions nest the case that data is aggregate using equal weights (flat aggregation). If $K = m$ (only current quarter information on the predictor is employed to forecast y_{t+h}), aggregation weights with a beta function nest a regression using only aggregate data when $\kappa = 1$, as is the case of $\kappa = 0$ with the exponential function. If p is the number of lags at the lower frequency, we can write $K = mp$. The notation can be then simplified by writing the weighted sum of $x_t^{(m)}$ as

$$x_{t(\kappa,mp)}^{(m)} = \sum_{j=1}^{mp} w(j,\kappa) L^{j/m} x_t^{(m)}, \tag{4}$$

such that a MIDAS regression is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,mp)}^{(m)} + \varepsilon_{t+h}.$$

2.2 Smooth Transition MIDAS

Switching regimes are a popular way of modelling nonlinear dynamics in regressions by using piecewise linear regimes linked by a transition function (Tong, 1990). When the transition between regimes is smooth and depends on the size of an observed transition variable, switching-regime models are called smooth transition regressions (surveyed by Van Dijk, Teräsvirta and Franses (2002)). When similar approach is applied to MIDAS regressions, regime switches depend on the size and sign of the weighted high frequency predictor.

The smooth transition MIDAS (STMIDAS) regression is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\lambda,mp)}^{(m)} \left[1 - G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c)\right] + \beta_{2,h}^{(m)} x_{t(\lambda,mp)}^{(m)} \left[G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c)\right] + \varepsilon_{t+h}, \qquad (5)$$

where

$$G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c) = \frac{1}{1 + \exp(-(\gamma/\hat{\sigma}_x)(x_{t(\alpha,mp)}^{(m)} - c)}.$$

The transition function $G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c)$ is a logistic function that depends on the weighted sum of the explanatory variable in the current quarter. The parameters of the function that weights the transition variable $x_{t(\alpha,mp)}^{(m)}$ may differ from the parameters weighting the predictor $x_{t(\lambda,mp)}^{(m)}$.

The function $G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c)$ has values between 0 and 1. When the smoothing parameter γ is large, the function is similar to an indicator function that is zero when $x_{t(\alpha,mp)}^{(m)} \leq c$ and equal to 1 when $x_{t(\alpha,mp)}^{(m)} > c$. Thus, the impact of $x_{t(\lambda,mp)}^{(m)}$ in predicting y_{t+h} is $\beta_{1,h}^{(m)}$ when the weighted sum of $x_t^{(m)}$ is small, and $\beta_{2,h}^{(m)}$ when the weighted sum $x_{t(\alpha,mp)}^{(m)}$ is large. When γ is small but is not equal to zero, the impact of $x_{t(\lambda,mp)}^{(m)}$ in predicting y_{t+h} is a time-variable weighted sum of $\beta_{1,h}^{(m)}$ and $\beta_{2,h}^{(m)}$ depending on the value of $G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c)$.

In order to check the restrictions required for identification of all the parameters, the STMIDAS regression is rewritten as:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\lambda,mp)}^{(m)} + \delta_h^{(m)} x_{t(\lambda,mp)}^{(m)} \left[G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c)\right] + \varepsilon_{t+h}, \qquad (6)$$

where $(\beta_{2,h}^{(m)} - \beta_{1,h}^{(m)}) = \delta_h^{(m)}$ in order to to simplify the notation. The assumption that $\sum_{j=1}^{mp} w\,(j; \lambda) = 1$ guarantees the identification of the slope parameter $\beta_{1,h}^{(m)}$, as in the case of MIDAS regressions. When imposing $\sum_{j=1}^{mp} w\,(j; \alpha) = 1$, the identification of γ and c are warranted. Finally, if in addition $\gamma > 0$, $\beta_{2,h}^{(m)}$ is identified. Similar restrictions are imposed to obtain identification of the parameters of the transition function in the flexible smooth transition regression of Medeiros and Veiga (2005).

The details of the application of nonlinear least squares to estimate STMIDAS regressions are in Appendix A.

This specification nests smooth transition regressions. When the parameters of the weight functions are such that the weighting function is flat ($\lambda = \alpha = 1$ for beta functions and $\lambda = \alpha = 0$ for exponential functions) and $p = 1$, the STMIDAS regression simplifies to a smooth transition regression (STR):

$$y_{t+h} = \beta_{0,h} + \beta_{1,h} x_t + \delta_h x_t \left[G_t(x_t; \gamma, c) \right] + \varepsilon_{t+h}. \tag{7}$$

An important advantage of the STMIDAS regression is that the delay of the transition variable does not need to be estimated/chosen when the transition variable is the weighted sum of past values. Becker and Osborn (2011) use a specification similar to STMIDAS regressions, but with aggregated regressors ($\lambda = 0$ or $\lambda = 1$ depending on the type of function), to test for nonlinearity.

Another feature of STMIDAS regressions is that they are designed for direct forecasting. Previous applications of non-linear time series models for verifying changes in the dynamic relationship between output growth and the spread (Galbraith and Tkacz, 2000; Anderson and Vahid, 2001; Galvão, 2006) have specified models only for one-step-ahead forecasts. Forecasts for longer horizons were then obtained by iteration with the aid of monte carlo methods to take into account nonlinearities on conditional expectations.

An alternative for modelling switching regimes is to make the regimes dependent on a latent variable that is controlled by a Markov process, as suggested by Guerin and Marcellino (2010). In comparison with this alternative, STMIDAS regressions have a regime-switching behaviour that depends on the size and sign of an observable variable available at high frequency. Therefore, STMIDAS regressions are able to capture asymmetries in the predictive content of $x_t^{(m)}$ to y_{t+h}. Galbraith and Tkacz (2000) argue that the slope of the yield curve has only predictive content for future output growth when the spread is small or negative. This kind of asymmetry can be easily captured by a STMIDAS model.

2.2.1 Inclusion of an Autoregressive Term

When computing out-of-sample forecasts, it is likely that an autoregressive term may improve forecasts as in the case of Ang et al. (2006). The STMIDAS specification with an autoregressive term is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\lambda,mp)}^{(m)} + \delta_h x_{t(\lambda,mp)}^{(m)} \left[G_t(x_{t(\alpha,mp)}^{(m)}; \gamma, c) \right] + \rho_h y_t + \varepsilon_{t+h}. \quad (8)$$

Clements and Galvão (2008) discuss the problem of including autoregressive terms in MIDAS regressions. The inclusion of only one autoregressive lag, as in equation (8), is adequate to the empirical applications in section 3.

2.3 Monte Carlo Evaluation

Monte Carlo simulations are employed for evaluating the properties of nonlinear least squares (NLS) for estimating STMIDAS regressions. In addition, this evaluation includes the measurement of forecasting performance of STMIDAS regressions against nested alternatives such as linear, MIDAS and smooth transition regressions, for different values of m, $\beta_{1,h}^{(m)}$ and δ_h.

The data generating process (DGP) is:

$$x_t^{(m)} = 0.05 + 0.98 x_{t-(1/m)}^{(m)} + \varpi_t;\ \varpi_t \sim N(0,1). \quad (9)$$

$$y_{t+1} = 0.5 + \ \beta_{1,1} \left(\textstyle\sum_{j=1}^{m} w(j,\lambda) x_{t-(j/m)}^{(m)} \right)$$

$$+\delta_1 \left(\textstyle\sum_{j=1}^{m} w(j,\lambda) x_{t-(j/m)}^{(m)} \right) G \left(\textstyle\sum_{j=1}^{m} w(j,\alpha) x_{t-(j/m)}^{(m)}; 6, 2.3 \right) + \varepsilon_{t+1}; \varepsilon_{t+1} \sim N(0,1)$$

The DGP of $x_t^{(m)}$ is based on the empirical estimates computed with the spread between 10-year and 3-month interest rates. The effect of sign-noise ratio on the estimation is assessed by comparing $\beta_{1,1} = 1.5$ (high) with $\beta_{1,1} = 0.5$ (low) as in Andreou et al. (2010). The R^2 of STMIDAS regressions with $\beta_{1,1} = 1.5$ are around 95%, and it is reduced to 70% when $\beta_{1,1} = 0.5$. The impact of the degree of nonlinearity on STMIDAS estimation is measured by comparing $\delta_1 = -.9$ (high) with $\delta_1 = -.3$ (low). Both aggregating functions are beta functions with two parameters (eq. 2) with $p = 1$, so $K = m$. The threshold c is set to 2.3, which is near the unconditional mean of the $x_t^{(m)}$ process, and the transition function is smooth ($\gamma = 6$). The parameters of the beta functions (λ and α) are set to mimic the empirical estimates computed with the spread as predictor of US output growth. The functions are plotted in figure 1 for $m = 13$ ($\approx$ weekly data) and $m = 65$ ($\approx$ daily data). The predictor's weighting function has an inverted U-shape, and the transition variable's weighting function is decreasing.

2.3.1 Biases and Efficiency of NLS estimation of STMIDAS regressions

The first monte carlo exercise aims at evaluating the effect of the size of $\beta_{1,1}$ and δ_1 on the size of the bias of NLS estimates for small ($T = 50, 100$) and large ($T = 500$) sample sizes. Table 1 presents results of four combinations of high/low sign-noise ratio and high/low nonlinearity. The values in Table 1 are median biases of NLS estimates across 5000 replications, and also 5% and 95% quantiles. These quantiles provide information on the efficiency of the estimates.

Instead of measuring biases of the weighting functions parameters (λ and α), I compute a measure of how well estimated weighting functions approximate the shape of true weighting functions, similar to the analysis of Ghysels and Valkanov (2006). The approximation errors are computed with the sum of the squared difference between the estimated and the true weighting functions, normalized by the squared weights of the true function:

$$\frac{\sum_{j=1}^{m} [w(j,\hat{\lambda}) - w(j,\lambda)]^2}{\sum_{j=1}^{m} w(j,\lambda)^2} + \frac{\sum_{j=1}^{m} [w(j,\hat{\alpha}) - w(j,\alpha)]^2}{\sum_{j=1}^{m} w(j,\alpha)^2} \tag{10}$$

The results in Table 1 suggest that biases and approximation errors decrease with the sample size. The small sample biases increase with decreasing degree of nonlinearity and sign-noise ratios. When $T = 500$, biases of transition function parameters are larger when $\delta = -.3$ in comparison with $\delta = -.9$. In very short samples ($T = 50$), the estimate of δ_1 is biased downwards (specially when in addition $\beta_{1,1}$ is small), indicating that it is hard to identify changes in the slope parameters. The effect of small sample sizes on forecasting performance is assessed in section 2.3.2.

The second exercise looks at the impact of estimating aggregation weights when the true model has flat weights (equivalent to $m = 1$). The true data generating process is a smooth transition regression.

The bias values when estimating the specification with flat aggregation (STR) are compared with biases when aggregation weights are estimated using either an exponential (eq. 3) or a beta (eq. 2) function. Table 2 presents the NLS estimation biases. Estimates obtained with the exponential function and a short sample ($T = 50$) exhibit γ and c biases that are larger than the ones with beta function. The threshold biases of assuming flat aggregation are larger than the ones of using beta aggregating function in small samples ($T = 50, 100$). If the sample size is large, estimation biases of assuming flat aggregation are equivalent to biases of using beta weighting function. However, quantile values indicate a narrower interval for γ when beta weighting functions are employed instead of assuming flat aggregation. This result suggests that the use of a transition variable sampled at high frequency improves estimation accuracy of transition function parameters. The results in Table 2 support the conclusion that there is no cost of estimating an aggregation function even when flat aggregation is adequate if the sample is large enough.

The third monte carlo exercise looks at the effect of assuming either flat aggregation ($m = 1$) or weekly aggregation ($m = 13$) when the true data generating process presumes predictors are sampled daily ($m = 65$) and aggregation is computed with beta functions. The high frequency predictor is described by an AR(1) process with large autoregressive coefficient; as a consequence, the results of Andreou et al. (2010) suggest that slope estimates are biased when the aggregating function has shape such as the one indicated in Figure 1 for $m = 65$. The interest of this exercise is to evaluate whether a mistaken aggregation scheme has also an impact on the parameters of the transition function (γ and r).

The effect of assuming flat aggregation ($m = 1$) on estimating transition function parameters is large in small samples (threshold bias is 1.6 for a true value of 2.3), and, even though it decreases with the sample size, it is still reasonable large when $T = 500$ (bias is 0.5). Note also that by assuming flat aggregation, γ biases are towards linearity ($\gamma = 0$). Also as a result of mistaken aggregation by using weekly data, the threshold bias is .13 when sample is short, and .07 when sample is large. The effect of threshold biases on slope biases are small ($m = 13$). In the next part, the repercussion of these biases on forecasting performance is measured. The results in Table 3 support the claim that mistaken aggregation (flat aggregation instead of using beta aggregation with $m = 65$) leads to large biases on the estimation of the threshold (c) and on the size of regime changes (δ).

2.3.2 Expected Relative Forecasting Performance of STMIDAS specifications

The empirical forecasting exercise will compare STMIDAS regressions with a linear regression (R), a MIDAS regression, and a smooth transition regression to assess forecast accuracy improvements from both nonlinearity and use of high frequency data. In this forecasting monte carlo exercise, the root mean squared forecast error (RMSFE) of STMIDAS one-step-ahead forecasts is compared with nested specifications: linear regressions, MIDAS regressions, and ST regressions. Similarly to previous exercises, the relative forecasting performance of STMIDAS models is exploited assuming different values of m, $\beta_{1,1}$ and δ_1, and sample sizes ($T = 50, 100, 500$).

Table 4 presents the RMSFE of the linear regression (R) and RMSFE ratios of alternative models with respect to the regression benchmark. At each column the data generating process is a STMIDAS (as described in eq. 9) with the indicated parameter values. Forecast results are presented using STMIDAS specifications with beta and exponential weighting functions.

MIDAS specifications provide large improvements in forecasting accuracy in comparison with the regression benchmark assuming a high sign-noise ratio ($\beta_{1,1} = 1.5$) and large difference between y and x frequencies ($m = 65$). Gains from nonlinearity when $\delta_1 = -.3$ are very small (0-5% of RMSFE) at the shortest sample. This suggests than when the nonlinearity is not very pronounced, a sample size of at least 100 observations is required to find reductions of around 5-7% of RMSFE. Gains from using disaggregate data on the predictor when sign-noise ratio is low are negligible (with $m = 13$). However, the impact of using high frequency data in a nonlinear specification (comparison of STMIDAS with STR) is sizeable even if $\beta_{1,1}$ is small. The use of the exponential weighting function instead of the DGP's beta function has almost no cost in terms of forecasting accuracy except when the sample is short ($T = 50$) and $m = 1$, 65. Finally, the use of $m = 13$ ($\approx$ weekly data) instead of $m = 65$ ($\approx$ daily data) worsens forecast accuracy for all T, but the increase in RMSFE is relatively small (up to 10% when $T = 50$).

In summary, these results suggest that forecasting gains from using STMIDAS regressions instead of a linear regression increase with the sample size, the degree of nonlinearity, the sign-noise ratio, and the true value of m.

3 Out-of-sample forecasting of US and UK output growth using financial predictors

This out-of-sample exercise compares the out-of-sample performance of STMIDAS regressions, which allow for asymmetries in the impact of the financial indicator on the future output growth, with regressions with symmetric effects such as linear and MIDAS regressions. This exercise also aims at evaluating the effect of estimating aggregation weights for financial data originally sampled daily and weekly. An important contribution of this exercise is to assess the impact of using mixed frequency data when dealing with smooth transition regressions. Finally, a fluctuation test of out-of-sample predictive ability is employed to check if financial indictors are capable of predicting output growth, while the forecasting model may be a linear, a MIDAS, a smooth transition or a STMIDAS regression.

In the remaining of this section, I describe data on financial indicators and output growth (section 3.1), the results on the out-of-sample relative forecasting performance from nonlinearities and use of high frequency data (section 3.2), and the results of out-of-sample predictive ability tests (section 3.3).

3.1 Data

Following the literature (for example, Estrella and Hardouvelis (1991)), the target variable of the predictive regression is $y_{t+h} = (400/h)\left[z_{t+h} - z_t\right]$, where z_t is the log-level of real GDP. The design of the forecasting exercise differs from Stock and Watson (2003) pseudo-out-of-sample exercise by making use of quarterly vintages of data available in real-time datasets.[4] Quarterly vintages refer

[4]Real-time data on US real output are obtained from the Philadelphia Fed website (http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/) and on UK real output are from the Bank of England website (http://www.bankofengland.co.uk/statistics/gdpdatabase/). UK real-time data is

to the time series of data available in the month at the middle of the quarter. With both UK and US real-time datasets, the last value in a given quarterly vintage is a preliminary estimate of the real GDP observed in the previous quarter. Forecast errors are computed using data from the latest available vintage as actuals, since the aim is to forecast true values, which are revealed by the revision process.

The chosen financial predictors are a term spread, a short-rate, and stock returns. The US term spread is measured by the difference between 5-year treasury bond rate and the 3-month bond rate, while the 3-month bond rate is the short-rate. The qualitative results do not change if the long-rate is the 10-year interest rate. Spread with the 10-year interest rate has been considered by Estrella and Hardouvelis (1991), while with the 5-year rate is employed by Ang et al. (2006). UK term spread is measured with a long-term treasury bond rate (Datastream) and the 3-month treasury bond rate. Stock returns have been employed as leading indicators by Zellner, Hong and Min (1991) and Estrella and Mishkin (1998), and they are computed using the annual difference in the price index. For the US, the SP500 is employed, while FTSE100 is employed to measure stock prices for the UK. US daily and weekly interest rates are obtained from the FRED database[5], and stock prices are from Datastream sampled daily. UK data on financial indicators are obtained from Datastream at daily frequency with start date in 1985:Q1. Weekly data are obtained using the value of the last day of the week. Note that stock returns are computed as $sr_t^{(13)} = 100(\ln(p_t^{(13)}) - \ln(p_{t-52}^{(13)}))$ with weekly data. Quarterly data are obtained by averaging weekly data within a quarter.

US output growth forecasts are computed with observations from 1970:Q1, and forecast origins from 1989:Q3 up to 2007:Q2, using vintages published from 1989:Q4 up to 2007:Q3. UK output growth forecasts are computed with observations from 1985:Q1, and forecast origins from 1995:Q3 up to 2007:Q2 and requiring vintages. Forecast errors are computed with data from the 2009:Q1 vintage for both countries.

published in monthly vintages which are converted to quarterly by using the monthly vintage at the middle of the quarter.

[5] http://research.stlouisfed.org/fred2/.

3.2 The Relative Forecasting Performance of STMIDAS regressions

Table 5 presents RMSFEs of forecasting with a linear regression for each one of three predictors and for one-quarter-ahead and one-year-ahead forecast horizons ($h = 1, 4$). It also includes ratios of RMSFEs of alternative models against the benchmark (ratios smaller than one imply alternative models are more accurate than the benchmark). Alternative models considered are MIDAS, ST and STMIDAS regressions. All forecast models are estimated with one autoregressive term, as in eq. 8. The models are estimated for a specific h, and forecasts are computed directly conditional on last available observations at the forecast origin.

The choice of a weighting function has generally a small impact on forecasting performance, as evaluated in section 2.3.2, consequently results in Table 5 are computed with beta functions. Even though a STMIDAS specification (eq. 5) nests specifications with equal weighting functions for predictor and transition variable ($\lambda = \alpha$), including STR's flat aggregating functions, I also include results with the restriction $\lambda = \alpha$ imposed. This restriction may help in cases that the optimisation procedure finds hard estimating both weighting functions.

Table 5 presents results with data sampled weekly ($m = 13$) and daily ($m = 62$). Andreou et al. (2009) prefer daily data since it is the typical high frequency sampling of financial variables. However, results in section 2.3.2 suggest that mixed data sampling models with daily data may have equivalent forecast performance to models with weekly data even if the true DGP assumes daily data. Finally, forecasting models in Table 5 use only one quarter information on the indicator ($p = 1$). The robustness of the results to this assumption is checked by presenting relative RMSFEs with $p = 2$ and $p = 5$ in Table 6. Smooth transition regressions are estimated using the current quarter value as transition variable (equivalent to delay equal to 1).[6] Table 7 presents results with $p = 1$ and weekly data when the forecast target is the UK output growth.

The results in Table 5, 6 and 7 were computed by estimating forecasting models with increasing

[6]Becker and Osborn (2011) show how to use weighting functions to remove the problem of selecting the delay of transition variables. When $p > 1$, this is an issue for the STR but not for STMIDAS regressions since they nest Becker and Osborn (2011) specification.

sample sizes at each forecasting origin (recursive forecasting scheme). This implies that the initial US sample size has 79 observations and it is enlarged up to 151 observations. The UK sample size is shorter, varying from 42 up to 90 observations. The out-of-sample period is divided into subsamples of 24 quarters: three are available with US data and two with UK data. The out-of-sample period is split because the predictive ability of financial indicators may be changing over time (as it will be discussed in the next section). This may have an effect on the relative importance of nonlinearity and/or use of predictors sampled at high frequencies.

The main results for forecasting US output growth are: (i) sizeable reductions of RMSFE (at least 4%) from modelling asymmetries are detected for at least one subperiod for each one of the indicators in both forecasting horizons; (ii) the use of high frequency data improves forecasts of smooth transition regressions more frequently than it does linear regressions, even though advances are generally small; (iii) the use of daily instead of weekly data does not generally improve forecasts; and (iv) accuracy gains from nonlinearity and direct use of weekly data are larger when p increases, but this is generally caused by a deterioration of the accuracy of the benchmark model (R).

Some additional results are also interesting. The imposition of the restriction that aggregating functions on the predictor and the transition variable are the same ($\lambda = \alpha$) has either a small impact or deteriorates forecasts, including when predicting UK output growth. However, when computing forecasts with the short-rate, the imposition of this restriction improves STMIDAS forecasts. About (ST)MIDAS specifications with daily data, sizeable improvements in accuracy are only detected with the spread as predictor during the first out-of-sample period.

When forecasting UK output growth with the short-rate, sizeable reductions of RMSFE (at least 4%) in comparison with the benchmark are found when using either the STMIDAS or the STR to compute forecasts at both horizons. Otherwise, linear regression forecasts are accurate in comparison to alternative forecasting models.

Both Tables 5 and 7 suggest that significant improvements in forecasting accuracy are more likely to arise from the inclusion of asymmetric dynamics and nonlinearities than from the use of high frequency data on the predictor.

3.3 The instability of out-of-sample predictive ability

3.3.1 Fluctuation Test

The fluctuation statistic developed by Giacomini and Rossi (2010) tests for equal forecast accuracy even if there is instability in the relative predictive accuracy between two forecasting models. The test is based on a local measure of relative loss function. In this specific application, if the null hypothesis is rejected, the forecasting model with the predictor is more accurate forecaster than the autoregressive benchmark at least once during the out-of-sample period. An important by-product of the test is the construction of a measure of local relative forecasting performance that is useful to assess whether financial variables have predictive power in some specific points in time. Based on the squared loss function, the difference between the benchmark model (autoregressive model; AR) and the economic model (EM) loss functions is

$$\Delta L_{h,t} = (y_{t+h} - \hat{y}_{t+h,AR})^2 - (y_{t+h} - \hat{y}_{t+h,EM})^2 \text{ for } t = N+1, ..., P+R$$

where P is the number of observations in the out-of-sample period and N is the number of observations of the in-sample period. By disregarding the impact of autoregressive terms on sample availability, the total number of observations is $T = P + N + h$. The relative loss may be computed for subsamples of the out-of-sample period, that is, the local relative loss is:

$$M^{-1} \sum_{j=t-M/2}^{t+M/2-1} \Delta L_{h,j} \text{ for } t = N + M/2 + ... + T - M/2 + 1,$$

where M is the window size. The fluctuation test statistic standardises the local relative loss by the variance of the relative loss function of the full out-of-sample period, that is,

$$F_{t,h,M} = \hat{\sigma}^{-1} M^{-1/2} \sum_{j=t-M/2}^{t+M/2-1} \Delta L_{h,j},$$

where $\hat{\sigma}^2$ is the HAC estimator of σ^2 computed to capture the variance of $\Delta L_{h,t}$ over $t = N + 1, ..., P + N$. Giacomini and Rossi (2010) derive the distribution of this statistic and provide critical values. Their monte carlo exercise on the empirical size and power of the fluctuation test suggests to choose M such that $M/P \approx 0.3$. The one-sided critical value at 5% confidence when $M/P = 0.3$ is

2.77. This means that if $F_{t,h,M}$ cross the critical value during the out-of-sample period, the economic model is more accurate than the autoregressive model at least once. A graphical analysis of the local relative loss can be used to check periods in which the economic model is significantly more accurate. Note that there are no restrictions on the number of times that $F_{t,h,M}$ crosses the critical value: the economic model may be only more accurate than the autoregressive model in subsamples of the out-of-sample period.

An important assumption in order to apply Giacomini and Rossi test is that forecasting models are estimated with rolling samples of size N. If the division between out-of-sample and in-sample periods is as in section 3.2., $N = 79$ with US data and $N = 42$ with UK data. For equivalently short sample sizes, monte carlo results described in section 2.3 suggest that large differences across specifications are unlikely to be found unless both degree of nonlinearity (size of the change in the slope coefficient) and sign-noise ratio are large.

An important feature of this out-of-sample exercise that may affect the properties of fluctuation test is the use of real-time vintages. Clark and McCracken (2009) show that when data revisions are predictable, the distribution of standard tests of forecasting accuracy is affected. It is less clear the effect of data revisions on the distribution of test statistics that assume that we are comparing forecasting methods as the Giacomini and Rossi (2010) approach. Finally, I aim at forecasting final revised data, while Clark and McCracken (2009) have actuals extracted from successive vintages of data. Rossi and Sekhposyan (2009) have applied the fluctuation test when using real-time data in the computation of forecasts and final data on the computation of forecasting errors.

3.3.2 Empirical Results of the Out-of-Sample Predictability Test

Figures 2 and 3 present the local relative squared loss between the four specifications considered in this empirical exercise and the autoregressive model for each one of the predictors when forecasting US and UK output growth at $h = 1$ and $h = 4$. The figures also indicate the one-sided 5% critical value of the fluctuation test. All specifications include an autoregressive term and assume $p = 1$. (ST)MIDAS regressions are estimated with weekly data and beta aggregation function. As the comparison between Tables 5 and 6 suggests, relative forecasting performance is generally robust to assumptions on m and p. Based on the results of Table 5, the STMIDAS specification when forecasting US output growth with the short-rate is restricted such that $\lambda = \alpha$.

Figure 2 confirms previous empirical results in the literature: the spread looses predictive power during the 90's (Giacomini and Rossi, 2006), the short-rate has stronger predictive content than the spread up to 1999 (Ang et al., 2006), and stock returns have only predictive content for short horizons (Estrella and Mishkin, 1998). Figure 3 also provides a new empirical result: the spread is back as predictor of US output growth (confirming the enduring predictive power of the spread for recessions argued by Rudebusch and Williams (2009)).

The inference on the predictive power of the spread at one-quarter-ahead horizon changes with the inclusion of both high frequency data and regimes changes: there is evidence of predictive ability in the middle of 90's, which was not detected with alternative forecasting models. The use of STMIDAS regressions also significantly improves US one-quarter-ahead output growth forecasts with stock returns in the beginning of 90's. However, there is no evidence that regime changes improve forecasts obtained with the short-rate.

UK output growth forecasting results (Figure 3) also share the extensive evidence of changes in predictive ability of financial indicators. Figure 3 suggests that the inclusion of high frequency data has almost no effect on the accuracy of forecasts, while regime changes affect negatively forecasting accuracy. An exception is found on the use of the short-rate to anticipate one-year-ahead output growth: models with regime changes suggest predictive content in the 00's, while models with constant coefficients provide no evidence of predictive content.

Common wisdom correlates recent downturns (2001, 2008) with stock market crashes. Results in Figures 2 and 3 suggest that stock returns are able to predict next quarter's UK and US economic activity during the first boom/crash period (96-02:Q2) but not in the second. However, the evidence of episodic predictive ability suggest that an extension of the estimation period beyond 2007:Q2 may change the evidence of no predictive power from stock returns. In the 00's, the spread is the financial indicator that has predictive content for one-year-ahead US growth while the short-rate is the variable for one-year-ahead UK growth.

4 Concluding Remarks

When assessing the predictive content of financial variables for economic activity, researchers normally aggregated data observed at daily frequencies before estimating a forecasting model that takes the relation between the financial variable and the dependent variable as linear (Stock and Watson, 2003). This paper proposes a forecasting model that relaxes both these assumptions by directly using high frequency data while taking into account regime changes in the slope parameters.

A simulation exercise using STMIDAS regressions as data generating processes indicates that substantial forecasting improvements from applying STMIDAS instead of linear regressions are expected when the sample size, the degree of nonlinearity, the sign-noise ratio, and the sample frequency of the predictor (m) are large. An empirical out-of sample exercise considers real-time forecasts of US and UK output growth using daily financial indicators. Significant improvements in forecasting accuracy are more frequently observed from asymmetries in the relation between financial variables and future output growth than from the direct use of weekly/daily data. However, accuracy enhancements from estimating aggregation weights are detected more frequently in nonlinear than in linear specifications.

The application of STMIDAS regressions for computing forecasts provides us with interesting insights on the nature of the real-time predictive content of the spread, the short-rate, and stock returns for UK and US output growth. Forecasts generated with STMIDAS regressions offer new evidence on the predictive ability of the spread and stock returns for next quarter US output growth in the earlier 90's, and on the ability of the short rate for next year UK growth in the earlier 00's.

A Estimation of STMIDAS

Recall the STMIDAS regression:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\lambda,m)}^{(m)} + (\beta_{2,h}^{(m)} - \beta_{1,h}^{(m)}) x_{t(\lambda,m)}^{(m)} \left[G_t(x_{t(\alpha,m)}^{(m)}; \gamma, c) \right] + \varepsilon_{t+h}$$

The parameters are collected in the vector $\theta_h = \left[\beta_{0,h}^{(m)}, \beta_{1,h}^{(m)}, \beta_{2,h}^{(m)}, \lambda, \alpha, \gamma, c\right]'$. The nonlinear regression is written as:

$$y_{t+h} = m(x_t^{(m)}, \theta_h) + \varepsilon_{t+h}.$$

Assuming that the restrictions required for identification described in section 2.2. are imposed, the parameters are consistently estimated by minimizing the sum of squared residuals:

$$Q_T(\theta_h) = T^{-1} \sum_{t=1}^{T} (y_{t+h} - m(x_t^{(m)}, \theta_h))^2,$$

because the function $m(x_t^{(m)}, \theta_h)$ satisfies the identification and regularity conditions described in Hayashi (2000), ch. 7, proposition 7.4. Under additional conditions regarding the differentiability of $m(x_t^{(m)}, \theta_h)$ and the behaviour of the Hessian $h\left(\hat{\theta}\right)$, the NLS estimator $\hat{\theta}_h$ is asymptotically normal, so that $\sqrt{n}(\hat{\theta}_h - \theta_h) \longrightarrow^d N(0, h(\theta_h)^{-1} \Sigma h(\theta_h)^{-1})$.

The computation of estimates is simplified by concentrating the sum of the squared residuals function with respect to λ, α, γ, c; as a result, parameters in vector $\beta_h = \left[\begin{array}{ccc} \beta_{0,h}^{(m)} & \beta_{1,h}^{(m)} & \beta_{2,h}^{(m)} \end{array}\right]'$ are computed with the least squares formula:

$$\hat{\beta}_h = \left(\sum_{t=1}^{T} x_{t(\hat{\lambda},\hat{\alpha},\hat{\gamma},\hat{c})}^{(m)} x_{t(\hat{\lambda},\hat{\alpha},\hat{\gamma},\hat{c})}^{(m)\prime} \right)^{-1} \sum_{t=1}^{T} x_{t(\hat{\lambda},\hat{\alpha},\hat{\gamma},\hat{c})}^{(m)} y_t.$$

In practice, STMIDAS regressions use MIDAS regression estimates as initial values for λ_1 and λ_2. Conditional on λ_1 and λ_2, initial values for γ, c and also α_1 and α_2 are jointly computed by grid search. Then conditional on chosen initial values of γ, c and also α_1 and α_2, and additional grid search is performed for new initial values for λ_1 and λ_2. This last step is more important if $\beta_{2,h}^{(m)} - \beta_{1,h}^{(m)}$ is relatively large. The optimisation procedure (with BFGS) imposes constraints in γ $(500 < \gamma > 0)$ and in c (a value of the threshold is not smaller (larger) than the 5% (95%) quantile of the empirical distribution of $x_{t(\alpha,m)}^{(m)}$).

Nonlinear least squares are also employed to estimate MIDAS regressions (Clements and Galvão, 2008) and smooth transition regressions (Van Dijk et al., 2002).

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Table 1. NLS estimation biases of STMIDAS regression parameters assuming beta aggregating function and m=13.

DGP:	$\beta_0 = .5$, $\beta_1 = 1.5$, $\delta = -.9$, $\gamma = 6$, r =2.3						$\beta_0 = .5$, $\beta_1 = 1.5$, $\delta = -.3$, $\gamma = 6$, r =2.3					
Biases on:	β_0	β_1	δ	γ	r	Approx. Error	β_0	β_1	δ	γ	r	Approx. Error
	T = 50						T = 50					
$q^{.50}$	0.00	0.00	-0.01	2.11	0.11	0.08	-0.02	0.02	-0.03	34.11	0.56	0.48
$q^{.05}$	-0.43	-0.18	-0.34	-3.40	-1.41	0.00	-0.45	-0.16	-0.40	-3.92	-5.53	0.01
$q^{.95}$	0.41	0.27	0.22	101.42	1.55	1.15	0.41	0.33	0.17	239.44	5.25	1.56
	T = 100						T = 100					
$q^{.50}$	0.00	0.00	0.00	0.44	0.08	0.03	-0.01	0.00	-0.01	19.10	0.41	0.23
$q^{.05}$	-0.28	-0.11	-0.14	-2.43	-0.67	0.00	-0.31	-0.11	-0.21	-4.14	-3.86	0.01
$q^{.95}$	0.27	0.11	0.13	21.19	0.83	0.57	0.28	0.16	0.13	294.22	3.59	1.39
	T = 500						T = 500					
$q^{.50}$	0.00	0.00	0.00	-0.06	0.01	0.00	-0.01	0.00	0.00	1.16	0.20	0.04
$q^{.05}$	-0.11	-0.04	-0.05	-1.16	-0.27	0.00	-0.15	-0.05	-0.05	-2.80	-0.89	0.00
$q^{.95}$	0.12	0.04	0.05	1.82	0.28	0.08	0.11	0.04	0.06	234.99	1.50	1.43
DGP:	$\beta_0 = .5$, $\beta_1 = 0.5$, $\delta = -.9$, $\gamma = 6$, r =2.3						$\beta_0 = .5$, $\beta_1 = 0.5$, $\delta = -.3$, $\gamma = 6$, r =2.3					
Biases on:	β_0	β_1	δ	γ	r	Approx. Error	β_0	β_1	δ	γ	r	Approx. Error
	T = 50						T = 50					
$q^{.50}$	0.06	0.03	-0.04	2.63	0.02	0.94	-0.01	0.03	-0.04	61.18	0.58	1.05
$q^{.05}$	-0.40	-0.20	-0.90	-4.49	-4.77	0.10	-0.44	-0.13	-0.42	-3.94	-5.18	0.11
$q^{.95}$	0.68	0.76	0.24	130.46	1.70	6.79	0.42	0.35	0.14	358.14	4.95	2.56
	T = 100						T = 100					
$q^{.50}$	0.02	0.02	-0.02	0.42	0.02	0.41	0.00	0.01	-0.02	54.35	0.44	0.64
$q^{.05}$	-0.27	-0.10	-0.46	-4.45	-3.90	0.04	-0.29	-0.09	-0.22	-4.01	-3.45	0.04
$q^{.95}$	0.38	0.39	0.13	33.72	0.91	6.25	0.30	0.17	0.11	492.78	3.22	1.96
	T = 500						T = 500					
$q^{.50}$	0.01	0.00	0.00	0.01	0.03	0.07	0.00	0.00	0.00	1.09	0.20	0.17
$q^{.05}$	-0.11	-0.04	-0.06	-1.33	-0.28	0.01	-0.12	-0.04	-0.06	-2.58	-0.74	0.02
$q^{.95}$	0.13	0.05	0.04	1.84	0.30	0.28	0.11	0.05	0.05	180.16	1.13	1.35

Note: $q^{.50}$ is the median bias, and $q^{.05}$ and $q^{.95}$ are the 5% and 95% quantile of the bias distribution over 5000 replications. DGP's weighting functions are described in Figure 1.

Table 2: NLS estimation biases of STMIDAS regression parameters assuming flat aggregating function and m=13.

	DGP:	$\beta_0 = .5,$	$\beta_1 = 1.5,$	$\delta = -.9,$	$\gamma = 6,$	$r = 2.3$
	Biases on:	β_0	β_1	δ	γ	r
Aggreg. Function:		T = 50				
flat	$q^{.50}$	-0.02	0.00	0.00	3.64	0.14
	$q^{.05}$	-0.54	-0.26	-0.43	-4.02	-3.02
	$q^{.95}$	0.55	0.34	0.30	192.30	1.57
exp.	$q^{.50}$	0.05	0.01	-0.01	18.44	-0.33
	$q^{.05}$	-0.56	-0.40	-0.62	-4.01	-4.74
	$q^{.95}$	0.80	0.51	0.41	158.68	2.08
beta	$q^{.50}$	0.02	0.01	-0.02	18.54	-0.02
	$q^{.05}$	-0.56	-0.29	-0.55	-3.89	-3.55
	$q^{.95}$	0.66	0.44	0.33	157.47	1.87
		T = 100				
flat	$q^{.50}$	-0.02	0.00	0.00	0.59	0.06
	$q^{.05}$	-0.36	-0.13	-0.19	-3.30	-1.35
	$q^{.95}$	0.35	0.15	0.17	53.44	0.94
exp	$q^{.50}$	0.05	0.01	-0.02	2.63	-0.36
	$q^{.05}$	-0.35	-0.14	-0.25	-3.48	-2.98
	$q^{.95}$	0.50	0.20	0.17	135.17	0.95
beta	$q^{.50}$	0.01	0.01	-0.01	2.87	-0.01
	$q^{.05}$	-0.37	-0.13	-0.23	-3.20	-2.17
	$q^{.95}$	0.47	0.18	0.17	101.25	1.05
		T = 500				
flat	$q^{.50}$	0.00	0.00	0.00	-0.08	-0.01
	$q^{.05}$	-0.14	-0.05	-0.06	-1.00	-0.50
	$q^{.95}$	0.15	0.05	0.06	4.00	0.42
exp	$q^{.50}$	0.04	0.01	-0.01	-0.68	-0.29
	$q^{.05}$	-0.14	-0.05	-0.08	-2.35	-1.30
	$q^{.95}$	0.22	0.06	0.06	1.64	0.32
beta	$q^{.50}$	0.00	0.00	0.00	-0.07	0.01
	$q^{.05}$	-0.15	-0.04	-0.07	-1.73	-0.53
	$q^{.95}$	0.15	0.05	0.06	2.58	0.42

Note: $q^{.50}$ is the median bias, and $q^{.05}$ and $q^{.95}$ are the 5% and 95% quantile of the bias distribution over 5000 replications.

Table 3: NLS estimation biases of STMIDAS regression parameters assuming beta aggregating function and m=65.

	DGP:	$\beta_0=.5,$	$\beta_1=1.5,$	$\delta=-.9,$	$\gamma=6,$	$r=2.3$
	Biases on:	β_0	β_1	δ	γ	r
Aggreg. Function:		T = 50				
flat	$q^{.50}$	-0.06	0.08	0.05	92.78	1.59
	$q^{.05}$	-1.10	-0.38	-1.11	-4.86	-5.81
	$q^{.95}$	1.23	0.96	0.51	494.00	5.48
exp	$q^{.50}$	0.01	0.00	0.00	-0.09	0.11
m=13	$q^{.05}$	-0.34	-0.17	-0.24	-3.54	-1.21
	$q^{.95}$	0.35	0.18	0.20	84.57	1.35
beta	$q^{.50}$	0.01	0.00	0.00	0.15	0.13
m=13	$q^{.05}$	-0.33	-0.15	-0.24	-3.52	-1.21
	$q^{.95}$	0.35	0.19	0.19	109.39	1.39
exp	$q^{.50}$	0.01	-0.01	0.00	0.81	0.00
m=65	$q^{.05}$	-0.31	-0.18	-0.21	-3.02	-1.06
	$q^{.95}$	0.39	0.16	0.19	42.34	0.93
beta	$q^{.50}$	0.01	0.00	0.00	0.77	0.01
m=65	$q^{.05}$	-0.32	-0.13	-0.21	-2.86	-0.90
	$q^{.95}$	0.32	0.16	0.16	70.04	0.92
		T = 100				
flat	$q^{.50}$	-0.03	0.09	0.00	23.65	1.31
	$q^{.05}$	-0.86	-0.30	-1.24	-5.32	-5.84
	$q^{.95}$	1.02	0.98	0.45	494.00	5.21
exp	$q^{.50}$	0.01	0.00	0.00	-0.95	0.10
m=13	$q^{.05}$	-0.22	-0.11	-0.14	-3.03	-0.72
	$q^{.95}$	0.24	0.11	0.13	22.84	0.90
beta	$q^{.50}$	0.01	0.00	0.00	-0.92	0.10
m=13	$q^{.05}$	-0.21	-0.10	-0.14	-3.03	-0.72
	$q^{.95}$	0.24	0.11	0.13	35.51	0.92
exp	$q^{.50}$	0.00	-0.01	0.01	0.02	0.00
m=65	$q^{.05}$	-0.21	-0.11	-0.11	-1.95	-0.51
	$q^{.95}$	0.23	0.09	0.13	7.05	0.49
beta	$q^{.50}$	0.00	0.00	0.00	0.18	0.01
m=65	$q^{.05}$	-0.21	-0.08	-0.12	-2.01	-0.48
	$q^{.95}$	0.21	0.09	0.10	7.77	0.49
		T = 500				
flat	$q^{.50}$	0.01	0.16	-0.14	-4.37	0.52
	$q^{.05}$	-0.34	-0.06	-0.83	-5.33	-3.20
	$q^{.95}$	0.39	0.64	0.15	-0.16	2.73
exp	$q^{.50}$	0.02	0.00	-0.01	-1.26	0.07
m=13	$q^{.05}$	-0.09	-0.04	-0.06	-2.27	-0.29
	$q^{.95}$	0.12	0.05	0.04	0.23	0.43
beta	$q^{.50}$	0.02	0.00	-0.01	-1.25	0.07
m=13	$q^{.05}$	-0.09	-0.04	-0.06	-2.26	-0.29
	$q^{.95}$	0.12	0.05	0.05	0.24	0.44
exp	$q^{.50}$	0.00	0.00	0.00	-0.28	-0.01
m=65	$q^{.05}$	-0.09	-0.05	-0.04	-0.99	-0.21
	$q^{.95}$	0.09	0.03	0.05	2.75	0.18
beta	$q^{.50}$	0.00	0.00	0.00	0.00	0.00
m=65	$q^{.05}$	-0.09	-0.03	-0.04	-0.96	-0.18
	$q^{.95}$	0.09	0.03	0.04	1.28	0.18

Note: $q^{.50}$ is the median bias, and $q^{.05}$ and $q^{.95}$ are the 5% and 95% quantile of the bias distribution over 5000 replications. DGP's weighting functions (m=65) are described in Figure 1.

Table 4: Relative forecasting performance of R, MIDAS, STR and STMIDAS when DGP is STMIDAS regression with beta aggregation function for m=13, 65 and flat aggregation for m=1.

DGP with:	m = 1		m=13				m = 65	
	$\beta_1 = 1.5$, $\delta = -.9$	$\beta_1 = 1.5$, $\delta = -.3$	$\beta_1 = 1.5$, $\delta = -.9$	$\beta_1 = 1.5$, $\delta = -.3$	$\beta_1 = 0.5$, $\delta = -.9$	$\beta_1 = 0.5$, $\delta = -.3$	$\beta_1 = 1.5$, $\delta = -.9$	$\beta_1 = 1.5$, $\delta = -.9$
	T = 50							
R	*1.74*	*1.11*	*2.16*	*1.59*	1.95	1.21	*3.12*	*3.12*
MIDAS (beta)	1.02	1.02	0.91	0.76	1.00	0.97	0.74	0.74
STR	0.64	0.96	0.77	1.01	0.67	0.95	0.95	0.95
STMIDAS (beta)	0.70	1.00	0.53	0.74	0.63	0.93	0.38	0.42(m=13)
STMIDAS (exp)	0.77	1.05	0.54	0.74	0.64	0.94	0.46	0.42(m=13)
	T = 100							
R	*1.76*	*1.11*	*2.18*	*1.59*	1.88	1.21	*3.07*	*3.07*
MIDAS (beta)	1.01	1.01	0.91	0.74	0.99	0.97	0.73	0.73
STR	0.59	0.92	0.73	0.96	0.65	0.92	0.94	0.93
STMIDAS (beta)	0.62	0.94	0.49	0.67	0.60	0.89	0.34	0.39(m=13)
STMIDAS (exp)	0.64	0.94	0.50	0.67	0.60	0.89	0.38	0.39(m=13)
	T = 500							
R	*1.77*	*1.11*	*2.09*	*1.59*	1.92	1.16	*3.06*	*3.06*
MIDAS (beta)	1.00	1.00	0.90	0.72	0.99	0.97	0.72	0.72
STR	0.57	0.91	0.72	0.95	0.62	0.91	0.91	0.91
STMIDAS (beta)	0.57	0.91	0.48	0.64	0.53	0.85	0.33	0.37(m=13)
STMIDAS (exp)	0.58	0.93	0.48	0.64	0.53	0.85	0.34	0.37(m=13)

Note: The entries for the linear regression (R) are the RMSFEs of one-step-ahead forecasts (computed based on average MSE across 5000 replications). The entries for the alternative models are ratios to the linear regression RMSFEs. The remaining coefficients do not change across columns: $\beta_0 = .5$, $\gamma = 6$, $r = 2.3$. Figure 1 shows weighting functions for m=13, 65.

Table 5: Forecasting Performance of MIDAS, STR and STMIDAS regressions relative to linear regression (R) when forecasting US output growth in real time.

Forecasting origin:	1989:Q3 - 1995:Q2	1995:Q3 - 2001:Q2	2001:Q3- 2007:Q2	1989:Q3 - 1995:Q2	1995:Q3 - 2001:Q2	2001:Q3 - 2007:Q2
MIDAS with:	Weekly data; beta weighting			Daily data; beta weighting		
Forecasting output growth at h=1						
	with spread					
R	2.243	2.541	2.077			
MIDAS	0.992	0.969	1.040	0.997	0.976	1.044
STR	1.009	0.966	1.013			
STMIDAS	**0.951**	**0.957**	1.035	**0.917**	0.971	1.035
STMIDAS ($\lambda=\alpha$)	1.047	0.965	0.992	1.039	0.984	1.015
	with short-rate					
R	2.277	2.243	2.389			
MIDAS	1.016	1.012	1.043	1.014	1.014	1.040
STR	1.079	1.014	**0.904**			
STMIDAS	1.154	1.064	1.064	1.150	1.090	1.077
STMIDAS ($\lambda=\alpha$)	**0.923**	1.051	**0.937**	0.990	1.048	0.975
	with stock returns					
R	2.110	2.047	1.900			
MIDAS	0.976	**0.955**	1.058	0.977	0.966	1.060
STR	1.031	0.992	**0.921**			
STMIDAS	1.021	0.981	1.168	0.990	0.998	1.200
STMIDAS ($\lambda=\alpha$)	1.022	0.982	1.169	0.968	0.995	1.202
Forecasting output growth at h=4						
	with spread					
R	1.480	2.068	1.206			
MIDAS	1.002	1.011	1.018	0.996	1.018	1.003
STR	0.904	0.991	1.031			
STMIDAS	**0.893**	1.002	1.018	**0.842**	1.018	1.021
STMIDAS ($\lambda=\alpha$)	0.990	1.006	0.993	**0.939**	1.017	0.978
	with short-rate					
R	1.508	1.609	1.621			
MIDAS	1.010	1.003	1.052	1.010	1.002	1.048
STR	1.497	**0.936**	1.237			
STMIDAS	1.424	**0.988**	1.295	1.321	0.980	1.325
STMIDAS ($\lambda=\alpha$)	1.341	0.986	1.315	1.460	**0.944**	1.311
	with stock returns					
R	1.589	1.741	0.983			
MIDAS	1.024	1.040	1.034	1.033	1.069	1.020
STR	1.174	**0.854**	1.005			
STMIDAS	1.148	**0.860**	**0.956**	1.277	**0.930**	1.017
STMIDAS ($\lambda=\alpha$)	1.184	**0.909**	**0.935**	1.185	**0.924**	1.164

Notes: Forecasts are computed using only the vintage available at each forecast origin (one quarter later than the dates indicated) and with increasing samples (recursive forecasting scheme). Forecast errors are computed using output growth data from the 2009:Q1 vintage as actual values. Ratios indicating RMSFE reductions larger than 4% are emboldened. All specifications with p=1.

Table 6: Forecasting Performance of MIDAS, STR and STMIDAS regressions relative to linear regression (R) when forecasting US output growth in real time: robustness check with weekly data and p=2, 5.

Forecasting origin	1989:Q3 - 1995:Q2	1995:Q3 - 2001:Q2	2001:Q3- 2007:Q2	1989:Q3 - 1995:Q2	1995:Q3 - 2001:Q2	2001:Q3 - 2007:Q2
Regressions with:	p = 2			p = 5		
Forecasting output growth at h=1						
	with spread					
R	2.247	2.548	2.075	2.143	2.587	2.089
MIDAS	0.987	0.972	1.044	1.019	0.965	1.035
STR	1.014	0.967	1.014	0.982	0.986	1.001
STMIDAS	**0.918**	**0.958**	1.042	**0.942**	**0.931**	1.024
	with short-rate					
R	2.223	2.328	2.254	2.315	2.264	2.381
MIDAS	1.052	0.977	1.105	0.974	0.992	1.015
STR	**0.989**	**0.966**	**0.933**	**0.954**	**0.952**	**0.907**
STMIDAS	1.214	0.986	1.181	1.112	0.996	1.171
STMIDAS (λ=α)	**0.927**	0.989	0.999	**0.920**	**0.974**	**0.930**
	with stock returns					
R	2.036	2.002	1.961	2.080	2.225	2.187
MIDAS	1.012	0.977	1.024	0.983	**0.887**	**0.920**
STR	1.040	1.043	1.080	1.048	1.115	1.196
STMIDAS	1.071	1.004	1.134	1.018	0.983	0.990
Forecasting output growth at h=4						
	with spread					
R	1.464	2.069	1.202	1.455	2.159	1.303
MIDAS	1.024	1.005	1.020	1.007	0.955	**0.932**
STR	0.933	0.991	1.025	0.975	0.996	0.962
STMIDAS	**0.885**	0.997	1.015	**0.881**	0.956	**0.928**
	with short-rate					
R	1.531	1.619	1.516	1.908	1.526	1.752
MIDAS	1.024	1.002	1.142	**0.856**	1.071	0.999
STR	1.358	0.998	1.247	1.182	1.106	1.081
STMIDAS	1.190	**0.975**	1.600	1.418	1.007	1.414
STMIDAS (λ=α)	1.245	**0.965**	1.411	1.088	1.057	1.199
	with stock returns					
R	1.503	1.746	1.100	1.481	1.836	1.153
MIDAS	1.082	1.091	0.975	1.095	1.051	**0.942**
STR	1.167	**0.819**	1.099	1.290	1.163	1.200
STMIDAS	1.266	1.004	1.172	1.474	**0.860**	1.002

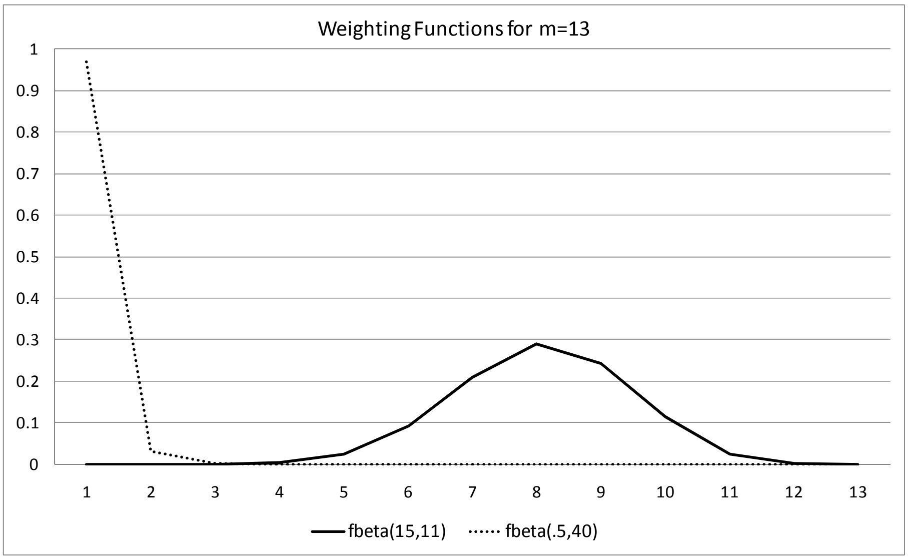
Notes: Forecasts are computed using only the vintage available at each forecast origin (one quarter later than the dates indicated) and with increasing samples (recursive forecasting scheme). Forecast errors are computed using output growth data from the 2009:Q1 vintage as actual values. Ratios indicating RMSFE reductions larger than 4% are emboldened.

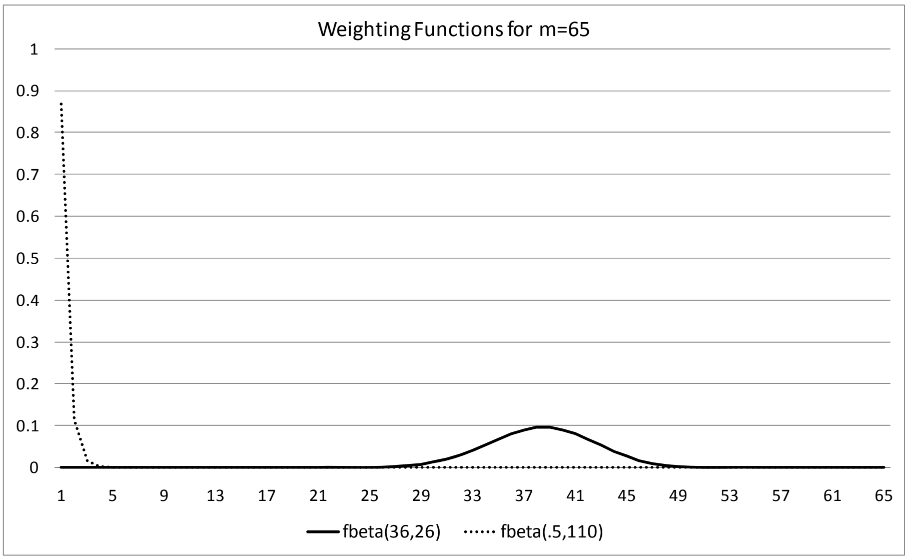
Table 7: Forecasting Performance of MIDAS, STR and STMIDAS relative to linear regression (R) when forecasting UK output growth in real time.

Forecasting origin:	1995:Q3 - 2001:Q2	2001:Q3-2007:Q2
Forecasting output growth at h=1		
	with spread	
R	1.538	1.126
MIDAS	1.000	1.000
STR	1.096	1.334
STMIDAS	1.103	1.342
STMIDAS (λ=α)	1.098	1.333
	with short-rate	
R	1.429	1.025
MIDAS	1.010	0.995
STR	**0.960**	1.026
STMIDAS	**0.962**	1.053
STMIDAS (λ=α)	**0.961**	1.047
	with stock returns	
R	1.109	1.073
MIDAS	0.998	1.010
STR	1.107	1.081
STMIDAS	1.305	0.982
STMIDAS (λ=α)	1.010	0.995
Forecasting output growth at h=4		
	with spread	
R	1.695	0.776
MIDAS	0.999	0.999
STR	1.502	2.051
STMIDAS	1.438	2.054
STMIDAS (λ=α)	1.460	2.049
	with short-rate	
R	1.152	0.591
MIDAS	1.005	0.978
STR	**0.915**	1.183
STMIDAS	**0.893**	1.223
STMIDAS (λ=α)	**0.893**	1.079
	with stock returns	
R	0.758	0.810
MIDAS	0.993	1.002
STR	0.968	1.592
STMIDAS	0.997	1.537
STMIDAS (λ=α)	0.993	1.550

Note: (ST)MIDAS regressions are computed with weekly data and beta weighting function. Forecasts are computed using only the vintage available at each forecast origin (one quarter later than the dates indicated) and with increasing samples (recursive forecasting scheme). Forecast errors are computed using output growth data from the 2009:Q1 vintage as actual values. Ratios indicating RMSFE reductions larger than 4% are emboldened

Figure 1: Weighting Functions of Data Generating Processes





Note: inverted U-shape functions aggregate data of the predictor ($w(\lambda)$) and increasing functions aggregate data of the transition variable ($w(\alpha)$).

Figure 2: Fluctuation test on the Predictive Ability of Financial Variables for US output growth (forecasting models estimated with rolling windows of 79 observations and test statistic computed with rolling windows of 24 observation; dates are the forecast origin in the middle of 24 obs. window).

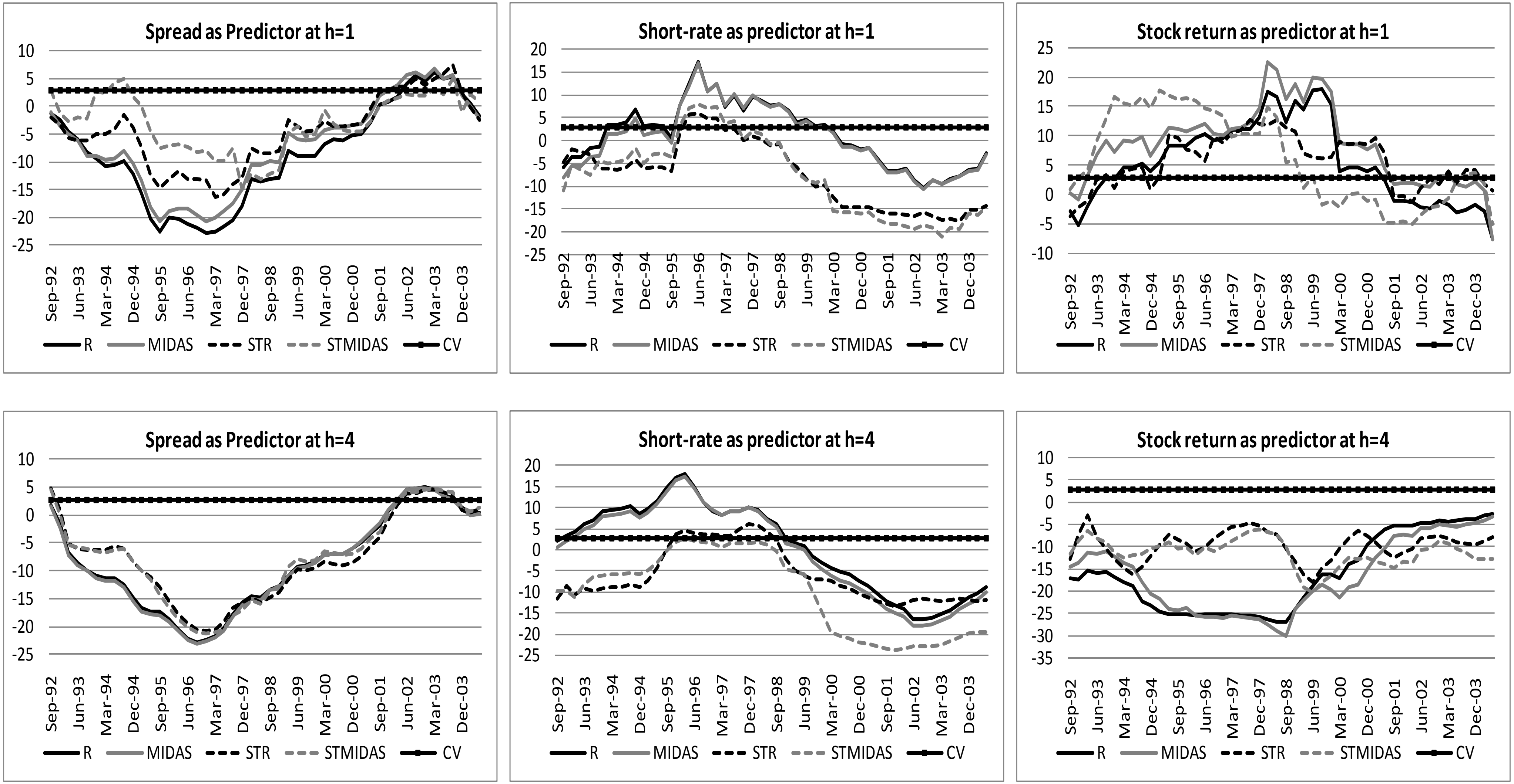


Figure 3: Fluctuation test on the Predictive Ability of Financial Variables for UK output growth (forecasting models estimated with rolling windows of 42 observations and test statistic computed with rolling windows of 24 observation; dates are the forecast origin in the middle of 24 obs. window).