International finance
Problem set 2

The change in the stock of net foreign assets in period 1 and 2 are respectively

$$B_1 - B_0 = rB_0 + Y_1 - C_1 \tag{1}$$
$$B_2 - B_1 = rB_1 + Y_2 - C_2. \tag{2}$$

Given finite horizon, solvency requires $B_2 \geq 0$ (the individual/country cannot die in debt). Furthermore, since the marginal utility of consumption is always positive $B_2 = 0$ (it is not optimal to die with a positive stock of assets).

Imposing $B_2 = 0$ in (2) and using it to replace for B_1 in (1) one obtains the intertemporal budget constraint (IBC)

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} + (1+r)B_0. \tag{3}$$

If $B_0 = 0$ PDV of lifetime consumption equals PDV of lifetime income. $B_0 > 0$ (positive wealth at birth allows higher PDV of consumption).

You are going to use (3) **both for question 1 and 3. Please go through questions 1 then 3 and finally 2 so that you can recycle your equations and save time.**

1. The (absolute value of the) marginal rate of substitution (how many unit of C_2 the consumer is willing to give up for a unit of C_1 along an indifference curve) is given by

$$MRS = \frac{MU_{C_2}}{MU_{C_1}} = \frac{\beta \frac{1}{C_2}}{\frac{1}{C_1}} = \frac{\beta C_1}{C_2}. \tag{4}$$

   (a) Since the country cannot trade, the good is non-storable and utility is strictly increasing, optimality requires endowments to be fully consumed in each period; i.e. $C_1 = Y_1$ and $C_2 = Y_2$. The autarky interest rate is such that $\frac{1}{1+r^A}$, the price ratio of C_2 in terms of C_1 equals the autarky MRS^A or (replacing for C_1, C_2 in (4) and equating to the price ratio)

$$\frac{\beta Y_1}{Y_2} = \frac{1}{1+r^A}, \tag{5}$$

   which implies

$$r^A = \frac{40(1+g)}{\beta 40} - 1 = \frac{1+g}{\beta} - 1 \sim .139. \tag{6}$$

   (b) Now the country can borrow and lend at the world rate r and is not forced to consume its endowment in every period. The market relative price of consumption today relative to tomorrow is now $\frac{1}{1+r}$ and at optimum it must equal the MRS in (4) or

$$\frac{\beta C_1}{C_2} = \frac{1}{1+r} \tag{7}$$

or

$$\frac{C_2}{C_1} = \beta(1+r). \tag{8}$$

The FOC determines the slope of the consumption profile. $C_2 \lesseqgtr C_1$ if $\beta(1+r) \lesseqgtr 1$. Since $\beta(1+r) = 1$ (8) becomes

$$C_2 = C_1. \tag{9}$$

One eq. in the two unknowns C_2, C_1. To obtain a unique solution we need a second equation: IBC (3).
Replacing for C_2 and rearranging we obtain

$$C_1 = \frac{1+r}{2+r}\left(Y_1 + \frac{Y_2}{1+r} + (1+r)B_0\right). \tag{10}$$

C_1 is roughly half (for r small) lifetime resources.
Replacing in (1)

$$CA_1 = S_1 = B_1 - B_0 = Y_1 + rB_0 - \frac{1+r}{2+r}\left(Y_1 + \frac{Y_2}{1+r} + (1+r)B_0\right). \tag{11}$$

Leave this equation on the board since you are going to use it for question 3.
In question 1 it is $B_0 = 0$ and (11) can be rewritten as

$$S_1 = B_1 - B_0 = \frac{1}{2+r}(Y_1 - Y_2) = \frac{1}{2+r}[40 - 40(1+g)] = -\frac{40g}{2+r}. \tag{12}$$

Since optimality requires a flat consumption profile (eq. (9)), saving is positive if $Y_1 > Y_2$ and negative viceversa. Since $g > 0$, the country is borrowing in the first period.

(c) It is

$$CA_1 = B_1 - B_0 = S_1 = -\frac{40g}{2+r} \tag{13}$$

where the last equality follows from (12).
Finally, $B_0 = 0$ and $B_2 = 0$ imply $CA_1 = B_1$ and $CA_2 = -B_1$. Hence, it is $CA_2 = -CA_1$. Solvency, requires the country to run a second period current account surplus equal (in absolute value) to the first period current account deficit.

(d) Y_2 increases relative to Y_1 and from (13), CA_1 falls to keep consumption smooth across time.

2. Assume for simplicity $B_0 = 0$ (the same result goes through with $B_0 \neq 0$). Balanced current account means $CA_1 = 0$ or $S_1 = B_1 = Y_1 - C_1 = 0$ and $CA_2 = Y_2 - C_2$.

Since the Israel economy is open to the rest of the world, for $CA_1 = 0$ the intertemporal optimality condition for consumption

$$\frac{C_2}{C_1} = \beta(1+r) \tag{14}$$

must be satisfied at $C_1 = Y_2$ and $C_2 = Y_2$; i.e. the autarky interest rate (MRS at the endowment point) equals the world one.

Denote by $Y_1' < Y_1$ the new, lower, level of first-period income. Since Y_2 is unchanged, (14) is no longer satisfied at the original endowment point. C_1 has fallen relative to C_2 at the original endowment point; i.e. the LHS of (14) increases. Since C_1 falls at the original endowment point, its marginal utility goes up. But optimality calls for the ratio of marginal utilities (and of consumption) in (14) to stay constant. This requires C_1 to exceed Y_1', the country runs a current account deficit in the first period to keep the ratio C_2/C_1 constant.

Alternatively, the autarky interest rate satisfies

$$MRS^A = \frac{\beta Y_1}{Y_2} = \frac{1}{1+r^A} = \frac{1}{1+r}. \tag{15}$$

$Y_1' < Y_1$ implies

$$\frac{1}{1+r} = \frac{\beta Y_1}{Y_2} > \frac{\beta Y_1'}{Y_2} = \frac{1}{1+r^{A\prime}} \tag{16}$$

or $r^{A\prime} > r$. After the shock, the country borrows at the world interest rate, since the latter is lower than the autarky one.

3. Notice that there was a typo. A debt reduction implies an *increase* in B_0 (a fall in absolute value).

   (a) IBC is (10) above. The optimality condition for consumption is (8) which, given $\beta(1+r) = 1$, reduces to (9).

   (b) The country's total income in the first period is $Y_1 + rB_0$. B_0 increases as a consequence of the debt reduction and so does total first period income.

   C_1 is given by (10) and also increases as the country PDV of lifetime resources increases.

   CA_1 is given by (11) with $Y_2 = 0$. Rearranging it can be written as

$$CA_1 = \frac{1}{2+r}\left[Y_1 + B_0\left(r(2+r) - (1+r)^2\right)\right] = \frac{1}{2+r}\left[Y_1 - B_0\right]. \tag{17}$$

   CA_1 falls in response to debt reduction (increase in B_0) as the country borrows to carry some of the increase in lifetime resources over to future consumption.