ECOM 009 Macroeconomics B

Lecture 3

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Predictions of the PICH

1. **Marginal propensity to consume out of wealth windfalls $\sim$ 0.03.** Roughly 0.3 in the data.
2. **$\boldsymbol{\Delta c_t}$ is an innovation (orthogonality).** Δc_t should be orthogonal to any variable in the consumer information set at time t. Early tests (Hall 1978) did not reject the joint hypothesis that $\beta_1 = \beta_2 = ... = \beta_k = 0$ in a regression of the type

$$c_t = \beta_0 c_{t-1} + \beta_1 y_{t-1} + \beta_2 y_{t-2} + ... + \beta_k y_{t-k} + e_t. \qquad (48)$$

Predictions of the PICH (ctd)

3. **$\mathbf{\Delta c_t}$ equals the innovation in permanent income.**

$$\Delta c_t = \theta \epsilon_t \tag{49}$$

where ϵ_t is the innovation in income process and θ is a function of the parameters of the income process. E.g. if

$$y_t = \mu + \lambda y_{t-1} + \epsilon_t \tag{50}$$

it is $\theta = r/(1 + r - \lambda)$.

Excess sensitivity: consumption responds too much to *predictable changes* in income.
Consider the equation

$$\Delta c_t = \beta \Delta y_t + \theta \epsilon_t. \tag{51}$$

- PICH coincides with the null hypothesis that $\beta = 0$.
- Conditioning on the innovation in permanent income, consumption must be uncorrelated with changes in income.
- If the hypothesis is rejected, consumption displays *excess sensitivity* with sensitivity parameter β.

Testing for excess sensitivity

Rewrite (50) as

$$\Delta y_t = \mu + (\lambda - 1)y_{t-1} + \epsilon_t. \tag{52}$$

Δy_t and ϵ_t are correlated, so we need to instrument Δy_t in (51). One possibility is using the income equation to instrument Δy_t to obtain

$$\Delta c_t = \beta(\mu + (\lambda - 1)y_{t-1} + \epsilon_t) + \theta\epsilon_t \tag{53}$$

$$= \beta\mu + \beta(\lambda - 1)y_{t-1} + (\beta + \theta)\epsilon_t. \tag{54}$$

It is clear that testing that $\beta = 0$ is equivalent to the orthogonality test conducted on (48).

Empirical studies

- Flavin (1981) rejected the hypothesis, finding a value of $\beta \sim .4$.
- Shea (1995), using data for unionized workers whose income growth is fairly predictable, finds that consumption growth is correlated with predicted income growth.
- This is important: if consumption is a random walk, it is effectively predetermined.
  - Policy changes (e.g. changes in taxes) have little effect on consumption unless they are permanent. In other words, the Keynesian multiplier is close to one. The Keynesian multiplier if roughly 1.7 if $\beta \sim .4$.

Breaking down the excess sensitivity puzzle

Recent evidence suggests that the excess sensitivity puzzle:

1. does not exist for **large**, anticipated income changes if one allows for preference non-separability (e.g. Browning and Collado 2003, Hsies 2003, etc.);
2. remains for relatively **small**, anticipated income changes, such as tax rebates, social security tax changes (Parker 1999, Souleles 1999, Johnson, Parker and Souleles 2003, etc.). The best candidate explanation is the existence of borrowing constraints.

(Lack of) excess sensitivity wrt to large income changes

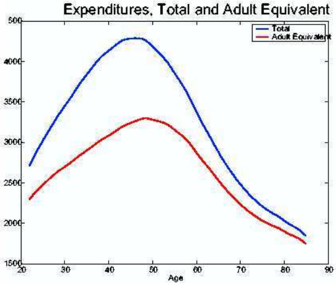

Figure : Source: Krueger 2005

1. Hump-shapedness: due to hump-shapedness of family size
2. Fall of consumption at retirement: substitutability between consumption and leisure

Family size

If per-period utility depends on per-capita consumption - i.e. $u(c_t/s_t)$ - with s_t family size, the Euler equation becomes

$$u'\left(\frac{c_t}{s_t}\right)\frac{1}{s_t} = \beta(1+r)E_t\left[u'\left(\frac{c_{t+1}}{s_{t+1}}\right)\frac{1}{s_{t+1}}\right]. \tag{55}$$

If $u(\cdot) = \frac{(\cdot)^{1-\sigma}-1}{1-\sigma}$, $\beta(1+r) = 1$ and there is no uncertainty the Euler equation can be rewritten as

$$\frac{c_{t+1}}{c_t} = \left(\frac{s_{t+1}}{s_t}\right)^{1-\frac{1}{\sigma}} \tag{56}$$

which implies

$$\frac{c_{t+1}}{c_0} = \left(\frac{s_{t+1}}{s_0}\right)^{1-\frac{1}{\sigma}} \tag{57}$$

If family size s_t is hump-shaped (it is!) so is consumption.

Consumption and leisure substitutability

Let now $s_t \in [0, 1]$ denote (exogenously given) leisure at time t. Assume again $\beta(1 + r) = 1$ and no uncertainty. If per-period utility is given by

$$u(c_t, s_t) = \frac{\left(c_t^{\gamma} s_t^{1-\gamma}\right)^{1-\sigma} - 1}{1 - \sigma} \tag{58}$$

the Euler equation can be written as

$$\frac{c_{t+1}}{c_t} = \left(\frac{s_{t+1}}{s_t}\right)^{\frac{(1-\gamma)(1-\sigma)}{1-\gamma+\gamma\sigma}} \tag{59}$$

which implies

Consumption and leisure substitutability II

$$\frac{c_{t+1}}{c_0} = \left(\frac{s_{t+1}}{s_0}\right)^{\frac{(1-\gamma)(1-\sigma)}{1-\gamma+\gamma\sigma}}. \tag{60}$$

- If $\sigma > 1$ and $\gamma \in (0, 1)$ the exponent is negative and increase in s_t/s_0 reduces consumption.
- Can explain fall of consumption at retirement.

Excess sensitivity wrt small income changes

Possible explanations

- A number of recent papers (Parker 1999, Souleles 1999, Johnson, Parker and Souleles 2003, etc.) have confirmed the excess sensitivity puzzle with respect to well-defined, predictable tax changes
- Possible explanations:
  1. liquidity constraints
  2. precautionary saving.

Excess smoothness

Excess smoothness: consumption responds too little to *innovations* (unpredictable changes) in income.

- PICH implies $\Delta c_t = \theta \epsilon_t$ and

$$\frac{\sigma_{\Delta c}}{\sigma_\epsilon} = \theta \qquad (61)$$

  with θ a function of the parameters of the stochastic income process.

- E.g. if $y_t = \mu(1-\lambda) + \lambda y_{t-1} + \epsilon_t$ it is

$$\theta = r/(1 + r - \lambda) \qquad (62)$$

Predictions vs facts

- In the data, aggregate consumption fluctuates less than income. In fact, this was one of the motivation for Friedman's Permanent Income Theory.
- In the data, e.g. Campbell and Deaton (1989), the ratio $\sigma_{\Delta c}/\sigma_{\epsilon}$ is in fact significantly less than one (~ 0.64.)
- Yet, equation (62) implies that the ratio predicted by the theory is smaller than one only if $\lambda < 1$ - output is trend-stationary.

Deaton Paradox

- **Deaton Paradox:** If income is not trend-stationary (as it appears to be the case) than the consumption response to income innovations displays *excessive smoothness* relative to the theory predictions.
- In particular, Deaton argues that the income process is best described by $\Delta y_t = \mu + \lambda \Delta y_{t-1} + \epsilon_t$ with $\lambda \sim 0.44$ which, together with $r = 0.01$ (quarterly), implies $\theta = (1 + r)/(1 + r - \lambda) \sim 1.77!$
- **Intuition:** theory predicts innovation in consumption equals innovation in permanent income. If income is highly persistent (difference stationary), an innovation in income implies an innovation in permanent income at least as great if not greater.

Possible explanations/solutions

Possible resolutions of the paradox:

1. variance of income process estimated by the econometrician may be higher than that of economic agents as the latter have information the econometrician does not observe. Consumption variance may not be excessively low relative to the true variance of income innovations.
2. Precautionary saving.

Point 1. is the standard inefficiency problem in econometrics when not all the information is used.

Recovering the true income variance

- Under rational expectations and if wealth is not subject to exogenous shocks (i.e. no capital gains/losses it can be solved) we can recover the true income variance even if the econometrician has a smaller information set than the consumer.
- Two possible solutions:
  1. use income variance from consumer survey data;
  2. use endogenous signalling variable to extract the extra (relevant) information available to the consumer.

Exploiting endogenous signalling variable (optional)

Consider the second possibility.

- Saving is chosen optimally after consumer observes current information.
- Saving reveals the (relevant) info available to the consumer.
- If the econometrician observes saving, we are done.

The saving equation again (optional)

Consider the saving for a rainy day equation:

$$s_t = -\frac{1}{1+r}\sum_{s=0}^{\infty}\frac{E(\Delta y_{t+s+1}|I_t)}{(1+r)^s} = -\sum_{s=1}^{\infty}\frac{E(\Delta y_{t+s}|I_t)}{(1+r)^s}. \qquad (63)$$

- It was derived using the consumption function, which embodies the PICH (optimization+RE+IBC).
- So equation (63) holds if and only if the PICH holds.
- Note that I_t in (63) is the consumer observation set.

Iterated expectations to the rescue (optional)

Suppose the econometrician information set is $\Omega_t \subset I_t$ but with $s_t \in \Omega_t$.

- Taking expectations of (63) with respect to Ω_t we obtain

$$s_t = E(s_t|\Omega_t) = -\sum_{s=1}^{\infty} \frac{E[E(\Delta y_{t+s}|I_t)|\Omega_t]}{(1+r)^s} \tag{64}$$

$$= -\sum_{s=1}^{\infty} \frac{E(\Delta y_{t+s}|\Omega_t)}{(1+r)^s}. \tag{65}$$

- The saving for the rainy day equation has to hold even if the income forecast is on the basis on a coarser information set as long as it includes the *signalling* variable saving.

Joint dynamics of income and saving (optional)

- Rather than estimating a univariate process for income we need to estimated the joint process for income and saving (i.e. condition on saving in estimating income).

$$\begin{bmatrix} \Delta y_t \\ s_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1_t} \\ u_{2_t} \end{bmatrix} \tag{66}$$

  with u_t an innovation with respect to past values of $\Delta y_t, s_t$.
- The univariate case in the previous section assumes $a_{12} = 0$ as past values of income carry all the information necessary to forecast current income.

Implications of orthogonality (optional)

- Take differences of the dynamic budget constraint $a_{t+1} = (1+r)a_t + y_t - c_t$ evaluated at t and $t-1$ respectively to obtain

$$s_t = (1+r)s_{t-1} + \Delta y_t - \Delta c_t. \tag{67}$$

- If the orthogonality restriction holds - equivalently if consumption does not display excess sensitivity - Δc_t is an innovation; i.e. $s_t = (1+r)s_{t-1} + \Delta y_t - e_t$ with $E(e_t|\Omega_{t-1}) = 0 \rightarrow E[s_t|\Omega_{t-1}] = (1+r)s_{t-1} + E[\Delta y_t|\Omega_{t-1}]$.

Orthogonality restrictions on the VAR (optional)

- Using (66) to form expectations, this holds true if

$$a_{21}\Delta y_{t-1} + a_{22}s_{t-1} = (1+r)s_{t-1} + a_{11}\Delta y_{t-1} + a_{12}s_{t-1} \quad (68)$$

or

$$a_{21} = a_{11} \text{ and } a_{22} = (1+r) + a_{12}.$$

- The above two restrictions are the **orthogonality restrictions.** If they are violated consumption is not a martingale and displays excess sensitivity to *predictable* income changes.

Orthogonality $\Leftrightarrow$ Lack of excess smoothness (optional)

- If consumption is an innovation, then integrating (67) implies the saving for a rainy day equation. **Orthogonality $\rightarrow$ Lack of excess smoothness.** The converse also holds. One is the mirror image of the other.
- If orthogonality fails, so does correct smoothness. Given a change in income, the IBC implies that lifetime income changes pin down lifetime consumption changes.
- As correct smoothness requires both the Euler equation (orthogonality) plus the IBC, if agents react excessively to forecastable income changes they must react too little to innovations in income for the IBC to be satisfied.

Possible explanations for failure of PICH

Liquidity constraints

Trivially explains excess sensitivity to predictable income changes.

The latter does imply excess smoothness given IBC.

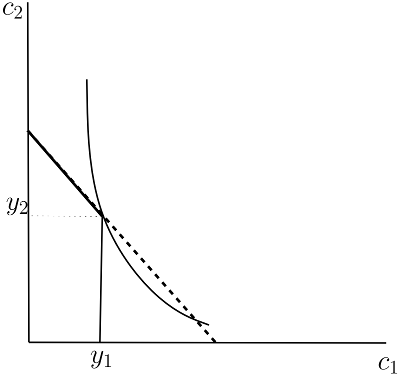

Figure : Liquidity constraints

Possible explations for the failure of PICH

Precautionary saving

- Quadratic felicity function → marginal utility is linear ($u''' = 0$), hence depends only on first moment of consumption.
- If $\beta(1+r) = 1$, flat expected consumption profile is necessary and sufficient for flat discounted expected marginal utility.
- If $u''' \neq 0$ Euler equation depends on further moments of consumption.
- In particular if $u''' > 0$, consumers dislike ex post consumption variability. Even if their expected consumption profile is flat, they save to self-insure against low income (and consumption) realizations.

Precautionary saving: prudence

- We keep assuming $\beta(1+r)=1$, hence the Euler equation

$$u'(c_t) = E_t[u'(c_{t+1)}] \sim \tag{69}$$

$$\sim E_t[u'(c_t) + u''(c_t)(c_{t+1} - c_t) + \frac{u'''(c_t)}{2}(c_{t+1} - c_t)^2]. \tag{70}$$

  After rearranging, this implies

$$E_t[c_{t+1} - c_t] = -\frac{u'''(c_t)}{u''(c_t)} \frac{E_t[c_{t+1} - c_t]^2}{2}. \tag{71}$$

- u''' is a sufficient (*but not necessary*) condition for precautionary saving. If consumption is stochastic consumers want an upward-sloping expected consumption profile with slope increasing in consumption uncertainty and in $-u'''(c_t)/u''(c_t)$.

Absolute and relative prudence

$$-\frac{u'''(c_t)}{u''(c_t)} \qquad \text{coefficient of absolute prudence}$$
$$-\frac{u'''(c_t)}{u''(c_t)c_t} \qquad \text{coefficient of relative prudence} \tag{72}$$

Precationary saving: a special case

Deriving a consumption function when consumers are prudent is impossible (only numerical solutions) with one exception:

- Negative exponential felicity function

$$u(c_t) = -\frac{1}{\gamma} e^{-\gamma c_t}. \tag{73}$$

- Income innovations normally distributed.
  Assume income process is

$$y_t = \lambda y_{t-1} + \epsilon_t \tag{74}$$

  with ϵ_t such that $\epsilon_t \sim N(0, \sigma)$.

Guess and verify

Finding a solution is not straightforward. We are going to guess a linear solution and verify that our guess is correct.

1. Guess a solution.

$$c_t = \alpha_0 + \alpha_1 a_t + \alpha_2 y_t \tag{75}$$

   with $\alpha_0, \alpha_1, \alpha_2$ unknown parameters to determine.
   The solution still implies that the consumption innovation is proportional to the income innovation.

2. Use the guess in the Euler equation and the dynamic budget identity and solve for the unknow coefficient.

Replacing in the Euler equation

The Euler equation is

$$e^{-\gamma c_t} = \mathbb{E}_t e^{\gamma c_{t+1}}$$

Replacing using our guess for the consumption function yields

$$e^{-\gamma(\alpha_0+\alpha_1 a_t+\alpha_2 y_t)} = \mathbb{E}_t e^{-\gamma(\alpha_0+\alpha_1 a_{t+1}+\alpha_2 y_{t+1})}$$

which can be rearranged as

$$e^{\gamma\alpha_1(a_{t+1}-a_t)} = \mathbb{E}_t e^{-\gamma(\alpha_2(\lambda y_t+\epsilon_{t+1})-\alpha_2 y_t)} = e^{\gamma\alpha_2(1-\lambda)y_t}\mathbb{E}_t e^{-\gamma\alpha_2\epsilon_{t+1}}$$

Manipulating the Euler equation

- If a r.v. $x \sim N(\mu, \sigma)$ then $\mathbb{E}e^x = e^{\mu + \frac{\sigma^2}{2}}$
- Applying to the above Euler equation yields

$$e^{\gamma\alpha_1(a_{t+1}-a_t)} = e^{\gamma\alpha_2(1-\lambda)y_t} e^{\frac{(\gamma\alpha_2\sigma)^2}{2}}$$

- Taking logs of both sides yields

$$\begin{aligned}(a_{t+1} - a_t) &= \frac{\gamma\alpha_2}{\gamma\alpha_1}(1-\lambda)y_t + \frac{(\gamma\alpha_2\sigma)^2}{2\gamma\alpha_1} \\ &= \frac{\alpha_2}{\alpha_1}(1-\lambda)y_t + \frac{\gamma(\alpha_2\sigma)^2}{2\alpha_1} \qquad (76)\end{aligned}$$

Replacing in the budget identlty

- Replacing for c_t in the dynamic budget identity yields

$$a_{t+1} - a_t = ra_t + y_t - \alpha_0 - \alpha_1 a_t - \alpha_2 y_t. \qquad (77)$$

- Equating the coefficients on the same variables on the RHS of (76) and (77) we obtain

$$\begin{aligned}
&\alpha_1 = r \\
&\frac{\alpha_2}{\alpha_1}(1-\lambda) = (1-\alpha_2) \;\Rightarrow\; \alpha_2 = \frac{r}{1-\lambda+r} \\
&\frac{\gamma(\alpha_2\sigma)^2}{2\alpha_1} = -\alpha_0 \;\Rightarrow\; \alpha_0 = -\frac{\gamma r \sigma^2}{2(1-\lambda+r)^2}.
\end{aligned}$$

Consumption and saving functions

$$
\begin{aligned}
c_t &= -\frac{\gamma r \sigma^2}{2(1-\lambda+r)^2} + r a_t + \frac{r}{1-\lambda+r} y_t \\
s_t &= \frac{\gamma r \sigma^2}{2(1-\lambda+r)^2} + \frac{1-\lambda}{1-\lambda+r} y_t \\
c_{t+1} - c_t &= r(a_{t+1} - a_t) + \frac{r}{1-\lambda+r}(y_{t+1} - y_t) \\
&= \frac{\gamma r^2 \sigma^2}{2(1-\lambda+r)^2} + \frac{r(1-\lambda)}{1-\lambda+r} y_t + \frac{r}{1-\lambda+r}(y_{t+1} - y_t) \\
&= \frac{\gamma r^2 \sigma^2}{2(1-\lambda+r)^2} + \frac{r}{1-\lambda+r} \epsilon_{t+1}
\end{aligned}
$$

Extra precautionary saving term

- Same consumption and saving functions as for PICH with AR(1) income process, but *extra* negative intercept for consumption and positive for saving (precautionary saving term).
- Income uncertainty σ and prudence γ increase *slope* of consumption profile and decreases *level*.

Implications of precautionary saving

1. Consumption is upward sloping. If income is expected to grow it can explain excess sensitivity to predicted income changes. It can explain why consumption is upward sloping (tracks income) for young people whose income is likely to be more uncertain.
2. If income uncertainty increases in old age (e.g. health shocks) it can explain low asset decumulation in old age.
3. If positive income innovations are associated with higher uncertainty about future income it can explain excessive smoothness.