ECOM 009 Macroeconomics B

Lecture 5

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Plan for this lecture

- To introduce the canonical, general equilibrium macroeconomic model with heterogeneous agents: the overlapping generation model with production.
- We are going to cover:
  - Environment, optimal choices, equilibrium.
  - Steady state equilibrium and transitional dynamic.
  - Aggregation: what determines aggregate saving.
  - Empirical evidence.
- Readings: Romer, Chapter 2 part B.

Introduction

- Up to now we have concentrated mostly on models in which agents are infinitely lived.
- Such models have the following implications/shortcomings:
  1. They cannot capture life-cycle issues.
  2. In general equilibrium, in the absence of uncertainty (cfr. Ramsey model)
     - they feature no trade as all agents either want to borrow or save at a point in time (lack of sufficient heterogeneity)
     - the decentralized competitive equilibrium is Pareto optimal (first welfare theorem applies).
- We now introduce a model of finite lifetimes which relaxes these limitations in a tractable way.

The overlapping generation (OLG) model

- Heterogeneous agents:
  - Young and old
  - Different endowments
  - Preferred model to study: life cycle problems (e.g. social security), market incompleteness, etc.
- Production economy.
- General equilibrium analysis: interest rate and labour income endogenously determined.

Economic environment

1. Time is discrete.
2. Demographics:
   - at each instant a new cohort of agents is born. The size of a cohort born at time t satisfies $L_t = (1+n)L_{t-1}$.
   - agents live for two periods $\rightarrow$ in each period a generation of young and one of old people coexist (heterogeneity).
3. Preferences: let c_t^i denote consumption, at time t, of an agent who is in period $i = 1, 2$ of her life at time t.
   - At time t young people maximize

$$u(c_t^1) + \beta u(c_{t+1}^2). \tag{102}$$

   - At time t old people maximize

$$u(c_t^2). \tag{103}$$

Economic environment II

4. Endowments: agents are born with no assets and have one unit of labour when young, zero when old.
5. Technology: CRS production function $Y_t = F(K_t, A_t L_t)$. Capital stock depreciates at rate δ. TFP evolves according to $A_{t+1} = (1 + g)A_t$.
6. Asset markets: the unique consumption good is storable and can be borrowed and lent at the riskless return r_t.
7. Market structure: competitive rental markets for capital and labour.

The consumer problem

A young consumer at time t solves

$$\max_{c_t^1, c_{t+1}^2, a_t^1} u(c_t^1) + \beta u(c_{t+1}^2) \tag{104}$$

$$\text{s.t. } a_{t+1}^1 = W_t - c_t^1,\ a_{t+2}^2 = (1 + r_{t+1}) a_{t+1}^1 - c_t^2,\ a_{t+2}^2 \geq 0. \tag{105}$$

The three constraints in (105) can be compacted into the IBC

$$c_t^1 + \frac{c_{t+1}^2}{1 + r_{t+1}} = W_t. \tag{106}$$

where we have imposed that it holds as an equality given $u' > 0$.

The consumer problem II

Therefore we can replace for either c_t^1 or c_{t+1}^2 in (104) and maximize to obtain the Euler equation

$$u'(c_t^1) = \beta(1 + r_{t+1})u'(c_{t+1}^2). \tag{107}$$

Together with IBC this allows to solve for c_t^1, c_{t+1}^2.

CRRA preferences

To get an explicit solution assume u is CRRA

$$u(c) = \frac{c^{1-\theta}}{1-\theta}, \ \theta > 0. \tag{108}$$

The Euler equation becomes

$$(c_t^1)^{-\theta} = \beta(1 + r_{t+1})(c_{t+1}^2)^{-\theta} \tag{109}$$

which implies

$$c_{t+1}^2 = [\beta(1 + r_{t+1})]^{\frac{1}{\theta}} c_t^1 \tag{110}$$

Replacing for c_{t+1}^2 in (106) one obtains the consumption function

$$c_t^1 = [1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}]^{-1} W_t. \tag{111}$$

Individual saving

- The saving function is

$$s_t^1 = a_{t+1}^1 - 0 = W_t - c_t^1 = \sigma(r_{t+1})W_t \tag{112}$$

with

$$\sigma(r_{t+1}) = 1 - [1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}]^{-1}. \tag{113}$$

- $\frac{\partial s_t^1}{\partial r_{t+1}} \gtreqless 0$ if $\theta \lesseqgtr 1$
- Income vs substitution effect.

Firms' problem

- Firms choose their capital and labour demands K_t^d, L_t^d so as to maximize profits

$$\max_{L_t^d, K_t^d} F(K_t^d, A_t L_t^d) - W_t L_t^d - (r_t + \delta) K_t^d \tag{114}$$

where r_t is the rental price of capital.

- Because of CRS we can write the production function in intensive form

$$\tilde{y}_t = \frac{F(K_t, A_t L_t)}{A_t L_t} = \frac{A_t L_t F(K_t / A_t L_t, 1)}{A_t L_t} = f(\tilde{k}_t). \tag{115}$$

Factor demands

The FOCs can then be written as

$$F_1(K_t^d, A_t L_t^d) = F_1\left(\frac{K_t^d}{A_t L_t^d}, 1\right) = f'(\tilde{k}_t^d) = r_t + \delta \tag{116}$$

and

$$A_t F_2(K_t^d, A_t L^d) = A_t[f(\tilde{k}_t^d) - f'(\tilde{k}_t^d)\tilde{k}_t^d] = W_t. \tag{117}$$

Market clearing

- Total supply of capital: total savings of the young

$$K_{t+1}^{s} = L_t a_{t+1}^{1} = L_t s_t^{1}. \tag{118}$$

- Total supply of labour: whole population of young agents

$$L_t^s = L_t. \tag{119}$$

- In equilibrium the two market clearing conditions

$$K_t^s = K_t^d \tag{120}$$

$$L_t^s = L_t^d \tag{121}$$

must hold.

Equilibrium

At any time t_0 an equilibrium is

- a sequence of allocations $\{c_t^1, c_t^2, L_t^d, L_t^s, K_t^d, K_t^s\}_{t=t_0}^{\infty}$; and
- factor prices $\{W_t, r_t\}_{t=t_0}^{\infty}$; such that
- (110)-(112), (116)-(121) are satisfied.

Equilibrium capital accumulation

- Replacing for s_t^1 in (118) using (112), (116) and (117) we obtain

$$K_{t+1} = L_t A_t \sigma(f'(\tilde{k}_{t+1}) - \delta)(f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t). \qquad (122)$$

- In intensive form

$$\begin{aligned}\tilde{k}_{t+1} &= \frac{A_t L_t}{A_{t+1} L_{t+1}} \sigma(f'(\tilde{k}_{t+1}) - \delta)(f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t) \\ &= \frac{1}{(1+g)(1+n)} \sigma(f'(\tilde{k}_{t+1}) - \delta)(f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t).\end{aligned}$$

Capital accumulation

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)}\sigma(f'(\tilde{k}_{t+1}) - \delta)(f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t) \quad (123)$$

- Non-linear, first-order difference equation which describes the evolution of the capital stock.
- Conceptually identical to the capital accumulation equations in the Solow and Ramsey models.
- As in the Ramsey model the saving rate out of labour income σ is endogenous.
- Depending on θ the equation may have multiple steady state equilibria. For this reason we now specialize to a particular technology and utility function.

Log utility and Cobb-Douglas technology

- Logarithmic utility: CRRA corresponds to log utility when $\theta = 1$.

  From equation (113), this implies

  $$\sigma = 1 - [1 + \beta^1 (1 + r_{t+1})^{1-1}]^{-}1 = \beta/(1+\beta). \tag{124}$$

  The individual saving rate is independent of the interest rate. Roughly half of first period endowment is saved (consumption smoothing).

- Cobb-Douglas technology: $F(K_t, A_t L_t) = K_t^\alpha (A_t L_t)^{1-\alpha} \rightarrow f(\tilde{k}_t) = \tilde{k}_t^\alpha,\ f'(\tilde{k}_t) = \alpha \tilde{k}_t^{\alpha-1}$.

Steady state

The capital accumulation equation (123) becomes

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)} \frac{\beta}{1+\beta} (1-\alpha) \tilde{k}_t^{\alpha}. \tag{125}$$

- Steady state: $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$.
- Two steady states: one with $k^* = 0$ and one with

$$\tilde{k}^* = \left[\frac{1}{(1+g)(1+n)} \frac{\beta}{1+\beta} (1-\alpha) \right]^{\frac{1}{1-\alpha}} \tag{126}$$

  - Higher g, n and lower β reduce $\tilde{k}^*$.
  - Lower $\beta \rightarrow$ lower saving rate. Same as Solow or Ramsey model.

Transitional dynamics

- Steady state with positive k^* is the only stable one.
  - $\tilde{k}_t$ converges to $\tilde{k}^*$ from right if $\tilde{k}_{t_0} > \tilde{k}^*$.
  - Viceversa if $\tilde{k}_{t_0} < \tilde{k}^*$.
- Speed of adjustment increasing in distance from steady state (conditional convergence).
- Increases in g, n and falls in β reduce off-steady state growth $\rightarrow$ Same as Solow growth model.

Transitional dynamics II

- Alternative way of looking at it.
- Divide both sides of (125) by $\tilde{k}_t$ and subtract 1 from both sides to obtain.

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t} - 1 = \frac{1}{(1+g)(1+n)} \frac{\beta}{1+\beta}(1-\alpha)\tilde{k}_t^{\alpha-1} - 1. \quad (127)$$

- As in Solow and Ramsey (exogenous growth models), convergence driven by decreasing returns to physical capital.