ECOM 009 Macroeconomics B

Lecture 6

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Aggregation

- The model feature heterogeneous agents (young and old) hence aggregation is non-trivial.
- Aggregate saving S_t equals total saving of the young plus the total saving of the old.
- Saving of an old agent at time t equals minus their saving when young (they eat all their assets by the end of their lifetime)

$$s_t^2 = r_t a_t^1 - c_t^2 = r_t a_t^1 - (1 + r_t) a_t^1 = -a_t^1.$$

- Hence (cfr. equation (118)) ,

$$S_t = K_{t+1} - K_t = L_t s_t^1 - L_{t-1} s_{t-1}^1. \tag{128}$$

Aggregate saving

We can use (**??**) and (122) to write

$$S_t = L_t A_t \sigma(f'(\tilde{k}_{t+1} - \delta)(f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t) - \tag{129}$$

$$L_{t-1} A_{t-1} \sigma(f'(\tilde{k}_t - \delta)(f(\tilde{k}_{t-1}) - f'(\tilde{k}_{t-1})\tilde{k}_{t-1}). \tag{130}$$

With log utility and Cobb-Douglas technology (129) becomes

$$S_t = L_t A_t \frac{\beta}{1+\beta}(1-\alpha)\tilde{k}_t^{\alpha} - L_{t-1} A_{t-1} \frac{\beta}{1+\beta}(1-\alpha)\tilde{k}_{t-1}^{\alpha} \tag{131}$$

$$= L_t A_t \frac{\beta}{1+\beta}(1-\alpha)\left[\tilde{k}_t^{\alpha} - \frac{1}{(1+g)(1+n)}\tilde{k}_{t-1}^{\alpha}\right]. \tag{132}$$

Aggregate saving II

$$S_t = L_t A_t \frac{\beta}{1+\beta}(1-\alpha)\left[\tilde{k}_t^{\alpha} - \frac{1}{(1+g)(1+n)}\tilde{k}_{t-1}^{\alpha}\right]. \tag{133}$$

Aggregate saving is positive if and only if the total saving of the young is larger than the total saving of the old.

Three possible reasons for this to happen.

- $g > 0$. Young get higher wages than old hence their saving is higher.
- $n > 0$. There are more young than old people. Total saving of the young exceeds the total dissaving of the old.
- Growing capital stock: $\tilde{k}_t > \tilde{k}_{t-1}$. Relevant only **off steady state**. Capital per efficiency unit and wages grow → saving of the young exceeds that of the old.

A list of saving motives

How can we obtain positive *aggregate* saving in general?

1. Consumption tilting: if $\beta(1+r) > 1$ (e.g. Ramsey model).
2. Precautionary saving: if there is income uncertainty and agents are either borrowing constrained with positive probability or prudent ($u''' > 0$) then they save.
3. Consumption smoothing: two cases to consider.
   - PICH. Saving for a rainy day. But if agents draw their labour income from the same distribution, by the law of large numbers, the saving of the lucky is offset by the dissaving of the unlucky. Aggregate saving is zero.
   - Life-cycle (the case here). Saving driven by fall in income at retirement.
     - Dissaving driven by consumption at retirement and borrowing by very young people in a model with more than 2 generations.
     - Positive aggregate saving is if total saving of savers exceeds the total dissaving of agents with negative saving.

What is the main driver of wealth accumulation?

- Important question: which of the above saving motives, if any, can explain the bulk of wealth accumulation?
- Tobin (1967) believed that life-cycle saving accounted for the bulk of wealth accumulation.

Empirical implications of life cycle model

- One of the prediction of the life cycle model is that the aggregate saving rate is increasing in the rate of growth.
- Aggregate saving rate:

$$\frac{S_t}{Y_t} = \frac{S_t}{A_t L_t \tilde{k}_t^{\alpha}}. \tag{134}$$

- Substituting for S_t using equation (131) in steady state it is

$$\frac{S_t}{Y_t} = \frac{\beta}{1+\beta}(1-\alpha)\left[1 - \frac{1}{(1+g)(1+n)}\right]\frac{(\tilde{k}^*)^{\alpha}}{(\tilde{k}^*)^{\alpha}} \tag{135}$$

$$= \frac{\beta}{1+\beta}(1-\alpha)\left[1 - \frac{1}{(1+g)(1+n)}\right]. \tag{136}$$

Empirical implications of the life cycle model II

- The model predicts a positive correlation between growth and saving rates.
- Cross country regressions of saving rates on growth rates give a coefficient of roughly 1.5.
- Yet, the prediction of the life cycle model comes through a clear mechanism: saving occurs at relatively early stages in life (i.e. it is done by people who have higher lifetime income, due to technological progress, and are in larger number.)
- In fact, across the lifetime consumption tracks income rather closely. Both are hump shaped, which implies that saving is rather flat rather than decreasing in age.

One test of the theory (Carroll and Summers 1991)

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- If we plot individual consumption against age, the higher the growth rate in a country the more downward sloping the consumption-age profile should be, as the younger have larger lifetime resources (life-cycle theory predicts that consumption depends on lifetime resources).
- Comparing the US and Japan household consumption over the period 1960-85 yields similar age consumption profiles despite the fact that the respective rates of growth were 2.1 and 5.2%. If anything the Japanese consumption age profile peaks later in life. The same holds true if one compares Thailand (4% growh rate) and Ivory Coast (0.9% growth rate).

One test of the theory II

- Household data do not take into account differences in demographic (family size across ages) composition across countries. Yet, the maximum size is obtained at roughly the same age in both countries.
- Without some differences in tastes (e.g. in β) across countries the model cannot explains the cross-country similarity. But differences in tastes are not what lies at the heart of the saving rate/growth correlation predicted by the life cycle model!

Efficiency of decentralized equilibrium

- The first welfare theorem tells us that, under some regularity conditions, a decentralized competitive equilibrium is Pareto optimal. This is indeed the case in the Ramsey model.
- This is not the case in the OLG model.
- The capital stock in the steady state equilibrium may be inefficiently high, so that it is possible to make everybody better off by reducing it.

Decentralized SS $\tilde{k}$

- For simplicity, assume zero TFP growth: $g = 0$.
- Remember that, in the log utility, Cobb-Douglas technology case, the steady state capital stock in efficiency units is given by

$$\tilde{k}^* = \left[\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\alpha)\right]^{\frac{1}{1-\alpha}} \tag{137}$$

The corresponding marginal product of capital is

$$\alpha(\tilde{k}^*)^{\alpha-1} = \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}(1+n). \tag{138}$$

Golden rule $\tilde{k}_{GR}$

- Compare

$$\alpha(\tilde{k}^*)^{\alpha-1} = \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}(1+n) \tag{139}$$

to the stock of capital which maximizes the total flow of consumable resource (Golden rule).
In efficiency units of labour this is given by

$$c^* = (\tilde{k}^*)^\alpha - (\delta+n)\tilde{k}^*. \tag{140}$$

Total output minus replacement investment.

- c^* is maximized at the Golden rule level of capital $\tilde{k}_{GR}$ satisfying the FOC of (140)

$$\alpha\tilde{k}_{GR}^{\alpha-1} = \delta + n. \tag{141}$$

Dynamic inefficiency

- For α close enough to zero, $\tilde{k}^* > \tilde{k}_{GR}$.
  - The decentralized equilibrium features too much capital and too low consumption as replacement investment is too high (dynamic inefficiency)
  - A one-off reduction in the capital stock from $\tilde{k}^*$ to $\tilde{k}_{GR}$ would yield a Pareto improvement.
- Why does the first welfare theorem fails?
  - One of the conditions for it to hold is that the number of optimizing agents is finite.
  - In Ramsey, the optimizing unit is the household and the number of households (though not their size) is finite.
  - Here, the number of optimizing agents coincides with the number of generations which is infinite.