**Macroeconomics B**
Problem set 3

*This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.*

1. Can finite lifetimes explain excess smoothness (Clarida 1991, Galì 1990). Suppose consumers maximize a quadratic utility function and can freely borrow and lend at a constant riskless rate $r$. Contrary to the permanent income model we assume consumers have finite lifetime. They start working as soon as they are born at age 1, work until age 40, retire at age 41 and die at the end of their 60th year. Both working life and retirement are deterministic. So a worker of age $t$ maximizes

$$\sum_{s=t}^{60} \beta^{(s-t)} u(c_s), \tag{6}$$

with $u(c_s) = -(c_s - \bar{c})^2/2$, and $\bar{c}$ large enough never to be reached. Suppose a worker's labour income process satisfies

$$y_t = y_{t-1} + \varepsilon_t, \tag{7}$$

with $\varepsilon_t$ white noise. A retired worker receives zero labour income. In other words

$$y_t = \begin{cases} y_{t-1} + \varepsilon_t & \text{if } t \leq 40 \\ 0 & \text{if } t > 40. \end{cases} \tag{8}$$

For simplicity assume $r = 0$ and $\beta = 1$.

(a) Write down the consumer sequence problem. Write down the Euler equation. Obtain the intertemporal budget constraint and solve for the consumption function for a consumer of arbitrary age $t$. Distinguish between retired and working consumers. (Hint: be careful when integrating the dynamic budget identity to obtain the intertemporal budget constraint. Lifetimes are finite. So, the appropriate solvency constraint is that terminal wealth cannot be negative).

(b) Derive the saving function for a consumer of arbitrary age $t$. Distinguish between retired and working consumers.

(c) Derive the relationship between the change in consumption $\Delta c_t = c_t - c_{t-1}$ and the innovation in labour income $\varepsilon_t$. This takes the form $\Delta c_t = k_t \varepsilon_t$. This is the response of consumption to an innovation in income for a consumer of age $t$. How is $k_t$ related to age $t$? What is its value at $t = 1$ and at $t = 40$?

(d) Assume that in every period there is a consumer of each age alive. Hence, the total population equals 60 individuals. Use a spreadsheet (e.g. Excel) and the value of $k_t$ calculated above to work out the average of $k_t$ in the population.

(e) What is the relationship between $\Delta c_t$ and $\varepsilon_t$ for the permanent income consumption model with infinite lifetimes and labour income following a random walk? How does this compare to the average $k_t$ you calculated above? Try explaining the difference between the two results.

2. Exercise 4 on page 41 in the chapter by Bagliano and Bertola. I am restating it here just for clarity. Assume the per period felicity function has the form $u(c) = c^{1-\gamma}/(1-\gamma)$ and be $\beta(1+r) \neq 1$. Derive the Euler equation for the consumer problem. Notice that $c_{t+1}/c_t = e^{\Delta \log c_{t+1}}$. Assuming $\Delta \log c_{t+1}$ has a normal distribution with unknown mean $E_t(\Delta \log c_{t+1})$ and known constant standard deviation $\sigma$ use the formulas in points 2. and 3. in the math handout I posted online to write down the Euler equation in terms of the unknown $E_t(\Delta \log c_{t+1})$ and $\sigma$. Solve for $E_t(\Delta \log c_{t+1})$ as a function of the model parameters. Comment on how it is affected by change in $\sigma$ and $\beta(1+r)$.