

Macroeconomics B

Problem set 6

This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.

This exercise is a simplified version of Lucas (1978).

Consider a two-period economy with many identical consumers. At time 1, consumers maximize the utility function

$$U(c_1, c_2) = \left(1 - \frac{1}{c_1}\right) + \frac{1}{1 + \rho} \mathbb{E}_1 \left(1 - \frac{1}{c_2}\right) \quad (9)$$

where c_1 and c_2 denote present and future consumption of coconuts.

Consumers have no initial financial wealth, but each owns a coconut tree whose current crop equals $Y_1 = Y$ coconuts. The next year crop is random according to the following distribution

$$Y_2 = \begin{cases} \frac{Y(1+g)}{(1-\sigma)} & \text{with probability } \frac{1}{2} \\ \frac{Y(1+g)}{(1+\sigma)} & \text{with probability } \frac{1}{2} \end{cases} \quad (10)$$

with $g, \sigma > 0$.

Consumers have to choose how much of the current crop to consume and how much to invest in a risk-free and in a risky asset. Denote by R_1^{-1} and P_1 the (ex-dividend¹) time-1 price, in terms of units of coconuts, of respectively one unit of the risk free and one unit of the risky asset. Denote by L_1 and N_1 the total number of units of the risk-free and risky-asset the consumers owns at the end of period t . Each unit of the risk-free asset pays one unit of coconut with probability one in period 2. On the other hand each unit of the risky asset pays a dividend Y_2 in period 2.

1. Write down the consumer dynamic budget identity in both periods and impose solvency. Note that we have wrote the problem in such a way that budget constraints look very similar to those in the lecture
2. At time t , the consumer chooses c_1, L_1, N_1 to maximize her expected lifetime utility, subject to the intertemporal budget constraint. Use the constraint to substitute out C_1 and derive the first order conditions for L_1 and N_1 .
3. Rewrite the above two first order conditions, to express R_1^{-1} and P_1 as a function of the consumer's stochastic discount factor.
4. Since coconuts are not storable and consumers are identical, in equilibrium prices must be such that consumers find it optimal to consume their endowment in every period; i.e. in equilibrium it has to be $c_1 = Y$ and $c_2 = Y_2$. Impose these equilibrium conditions in the first order condition and solve for prices as a function of the model parameters.

¹The ex-dividend price a share is the price after it has paid dividends for the current period. This is the same pricing convention used in the lecture notes.

5. Write down the ratio between P_1 and R_1^{-1} as a function of the model parameters. Discuss the economic intuition behind the value of the ratio.