**Macroeconomics B**

Problem set 7

*This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.*

Consider the problem of a firm which installs capital rather than renting it. Capital has unit price $p_t^K$ and depreciates at instantaneous rate $\delta$. The firm is subject to a proportional tax on profit $\tau$ and an enjoys an investment tax credit which subsidizes the price of one unit of investment at rate $\gamma$ and taxes the price of each unit sold at the same rate. All markets are competitive and the firm can freely borrow and lend at the instantaneous riskless rate $r$. There are no costs of adjusting either capital or labour and the capital stock cannot be negative.

All this implies that the firm's objective function is

$$\int_0^T (1-\tau)\left[A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - I_t p_t^K (1-\gamma)\right] e^{-rt} dt. \tag{11}$$

1. Write down the firm optimization problem and the Lagrangean associated with it (hint: follow the lecture notes for the case of adjustment costs keeping in mind that here the depreciation rate is positive).

2. Derive the first order necessary conditions for an optimum.

3. Note that the shadow price of capital is a function of $p_t^K$. Use the first order conditions to solve for the optimal capital/labour ratio as a function of $p_t^K$. Comment on how the optimality condition for the capital labour ratio compares with the Jorgensonian theory of investment.

4. How do $\tau$ and $\gamma$ affect the optimal/capital labour ratio.