5. The public sector budget constraint (infinite time)

In what follows we relax the assumption that the government lives for a finite number of periods. The assumption that the government is infinitely lived is a better approximation to reality and turns out to make a substantial difference.

The government budget identity (why is this an identity and not a constraint?) in nominal terms can be written (abstracting from some occational sources of cash such as privatisation revenues) as

$$B_t = G_t - T_t - \Delta M_t + (1 + i_t)B_{t-1}. \tag{1}$$

Dividing both sides by P_t we obtain the identity in real terms viz.

$$\frac{B_t}{P_t} = \frac{G_t - T_t}{P_t} - \frac{\Delta M_t}{P_t} + (1 + i_t)\frac{B_{t-1}}{P_t} \tag{2}$$

The price level increases at the rate of inflation: i.e. $P_t = (1 + \pi_t)P_{t-1}$. Then we can rewrite equation (2) as

$$\frac{B_t}{P_t} = \frac{G_t - T_t}{P_t} - \frac{\Delta M_t}{P_t} + \frac{1 + i_t}{1 + \pi_t}\frac{B_{t-1}}{P_{t-1}}. \tag{3}$$

Noticing that in discrete time Fischer equation is $1 + i_t = (1 + r_t)(1 + \pi_t^e)$ we can write

$$b_t = -s_t - \sigma_t + \frac{(1 + r)(1 + \pi_t^e)}{1 + \pi_t}b_{t-1} \tag{4}$$

where s_t is the real primary sector surplus, σ_t is real seignorage and b_t is the real stock of debt at time t and the real interest is assumed to be constant over time.

Equation (4) shows that for the real stock of debt to fall over time it has to be that either a) the primary surplus is positive, or b) seignorage revenue exceeds the primary deficit, or c) the government creates unexpected inflation or any combination of these. If inflation is perfectly anticipated ($\pi_t = \pi_t^e$) the third option is not viable and (4) becomes

$$b_t = -s_t - \sigma_t + (1 + r)b_{t-1}. \tag{5}$$

Equation (5) can be used to work out how the stock of debt will evolve for given r, b_{t-1} and the path of s_t and σ_t. If $r > 0$, then the stock of government debt increases at a rate no smaller than r unless the sum of seignorage revenue plus the primary surplus is positive.

1 Government solvency (infinite time)

If time is finite, solvency implies that agents die with a non-negative stock of debt. Clearly this definition is not very helpful for an agent who is infinitely lived. In infinite time, solvency requires an agent (in our case the government) to be able to meet its payments.

Consider equation (5). If $s_t + \sigma_t \leq 0$ then the stock of debt grows at a rate at least equal to r. This means that the government is paying interests by issuing new debt. Can this continue indefinitely? Or, using some jargon, can the government run a Ponzi game?

Equation (5) can be rearranged as

$$b_{t-1} = \frac{s_t + \sigma_t}{1+r} + \frac{b_t}{1+r}. \tag{6}$$

The government can place the current stock of debt b_{t-1} if either it repays it or it is able to roll it over at the end of the period. If the right-hand side of the above equation exceeds the left-hand side the government is bankrupt. But if agents are forward-looking they are willing to subscribe the current debt only if equation (6) holds at any future instant. Otherwise at some point in the future the government will default with certainty and nobody would hold the current stock of debt. Substituting recursively, equation (6) becomes

$$b_{t-1} = \frac{1}{1+r}\left(\sum_{i=t}^{T} \frac{s_i + \sigma_i}{(1+r)^{i-t}} + \frac{b_T}{(1+r)^{T-t}}\right) \tag{7}$$

Equation (7) can be used to work out the PDV of future surpluses plus seignorage consistent with a given level of debt at a future instant T for a given interest rate r and initial stock of debt $b_{t-1.}$

By letting T go to infinity equation (7) can also be used to discuss solvency when time is infinite. We can distinguish two cases:

1. $\lim_{T\to\infty} b_T/(1+r)^{T-t} = 0$. The PDV of the future stock of debt converges to zero; i.e. **on average** the stock of debt grows at a rate which is smaller than r. That is the government does not pay for **all** interests by issuing new debt. On average it pays out of current resources a positive fraction of the interests rB_t (i.e. $s_t + \sigma_t > 0$ on average). The government does not need to repay eventually ($b_T > 0$), yet the current stock of debt has to satisfy the intertemporal budget constraint (why is this a constraint and not an indentity?)

$$b_{t-1} = \sum_{i=t}^{\infty} \frac{s_i + \sigma_i}{(1+r)^{i+1-t}} \tag{8}$$

The government is solvent if and only if the current stock of debt does not exceed the PDV (over the infinite future) of future surpluses + seignorage.

2. $\lim_{T\to\infty} b_T/(1+r)^{T-t} > 0$. The government debt is increasing at a rate no smaller than r. Interests are paid by issuing new debt. Can the government still be solvent? That is, can it meet its obligations (rolling over its debt and pay interests)?

   The government is effectively running a pyramidal scheme. It is paying maturing debt by issuing new debt. Such a scheme is viable if and only if at any instant there are enough people willing to subscribe the new debt.

   Suppose GDP grows at a constant rate equal to g. We can distinguish two cases.

   - $g < r$. The stock of debt is growing at a rate which is faster than GDP. Since the full amount of interests rB_t is paid by issuing additional debt, the flow of new debt that needs to be placed also grows at rate r. As national saving cannot exceed GDP which grows at the lower rate g, in finite time the size of new debt issueances will exceed national saving (and income) and the government will not be able to place them.. Forward-looking agents forecast this and will not lend unless $\lim_{T\to\infty} b_T/(1+r)^{T-t} = 0$.

- $g > r$. As GDP grows faster than the stock of debt, eventually the latter will become a negligible proportion of GDP. So the government will always be able to roll over its debt even if it is running a Ponzi game. Agents are willing to hold the debt because it pays them the market interest rate and because the government will always pay them back by borrowing from the (richer) future generations. The government is solvent despite the fact that the current stock of debt exceeds the PDV of future surpluses. Incidentally, note that in this case Ricardian equivalence would not hold even if capital markets were perfect, agents infinitely lived and so on.

So **unless a country is so lucky that its rate of growth exceeds the real interest rate, if time is infinite solvency requires the budget constraint (8) to be satisfied.**

2 Summary of positive and normative issues in fiscal policy

This first series of lectures has explored the positive and normative theory of fiscal policy.

The main lessons to be learnt are:

1. Debt financed government expenditure may or may not crowd out private investment (it does for sure if it is wasteful, but it depends otherwise).

2. For a given path of government expenditure, debt financing as opposed to tax financing today is desirable if it allows for tax and consumption smoothing (both within and across generations).

   Yet, unless the rate of growth of the economy exceeds the real interest rate, debt financing can only reallocate taxes across time. For a given initial level of debt and given path of government expenditure, the PDV of taxes has to satisfy the solvency constraint (8).

3. Concerns about debt are often driven by fears that: a) excessive deficits will eventually result in a debt-deficit spiral in which interest payments grow explosively and the government defaults; b) the government may resort to excessive money printing (seignorage) in order to remain solvent. (e.g. if Italy runs excessive deficits will other EMU countries have to print too much money in order to bail it out?).