

**Macroeconomic Policy**  
**Exercise set 10**

1. (Optimal contract for central bankers) Consider the following problem. The economy is described by the SRAS curve

$$y_t = \bar{y} + \alpha (\pi_t - \pi_t^e) \quad (1)$$

where  $\alpha$  is a positive constant,  $y_t$  is the current output level and  $\bar{y}$  is its equilibrium, full-employment level.  $\pi_t$  is actual inflation and  $\pi_t^e$  is the private sector expectation of the inflation rate. Private sector expectations are rational and formed before the authorities determine actual inflation. Assume the economy lasts two periods only, so that there is no way for the central bank to establish a reputation. The government objective function is given by

$$W_G = y_t - \frac{\pi_t^2}{2}. \quad (2)$$

To lessen the time inconsistency problem the government appoints an independent central banker's whose objective function takes the form

$$W = y_t - \gamma \frac{\pi_t^2}{2}, \quad (3)$$

with  $\gamma > 1$ . You know from the lecture notes that the time-consistent rate of inflation in this economy is positive. To further reduce the extent of the time-inconsistency problem the government ties the central banker's pay to his/her inflation performance. The banker's pay is given by  $m = b + c\pi$  where  $b$  and  $c$  are parameters (not necessarily positive)

1. Intuitively, would you expect  $c$  to be positive or negative?
2. Given his/her contract the central bank now chooses  $\pi_t$  to maximize the sum of  $W$  in equation (3) plus income

$$W' = y_t - \gamma \frac{\pi_t^2}{2} + b + c\pi \quad (4)$$

subject to the constraint (1).

What is the time consistent rate of inflation? What are the values of  $b$  and  $c$  that maximize government welfare  $W_G$  (that is, we are implicitly assuming that the government does not care about how much it pays the central banker)?