Empirical Finance

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Lecture 01: Efficient Market Hypothesis

references
Lo & MacKinlay, chapter 1
Campbell, Lo & MacKinlay, chapter 1
Cuthbertson & Nitzsche, chapters 2 and 3
Kendall’s time-series results

business cycle economists use to believe in the 1950s that tracing the evolution of several economic variables over time would clarify and help predicting the progress of the economy through boom and bust periods

natural candidate: stock market prices for they reflect the prospects of the firms and hence peaks and troughs in economic performance ought to show up

Kendall (JRSS 1953): no predictive patterns in stock prices

→ animal spirits dominate the stock market
   (erratic market psychology)
Does such an interpretation hold water?  

if stock prices are predictable, then investors would reap **unending profits** simply by purchasing the stocks with positive forecasts and selling those stocks about to fall in price $\rightarrow$ not really!!!

**example**: if the stock price is about to increase by $\Delta\%$, all investors would like buy the stock, but no one would like to sell

**supply vs demand** $\rightarrow$ price would increase by exactly $\Delta\%$

$\rightarrow$ stock prices will **immediately reflect** the forecast news!!!

**intuition**: forecasts about the *future* performance determines instead *current* performance, as markets participants will attempt to get in on action before the price jump $\rightarrow$ **anticipation argument**
stock prices must reflect any **public information** that relates to stock performance (e.g., information on the macroeconomy or on the firm’s industry and management) \(\rightarrow\) **information flow vs trading activity**

\(\rightarrow\) stock prices then respond only to new information, which is by definition **unpredictable**, otherwise it would have been part of today’s information set

if stock prices react only to unpredictable news, they must also move in a random and unpredictable manner \(\rightarrow\) **random walk**
Classic interpretation

far from supporting market irrationality, randomly evolving prices are the necessary consequence of rational investors competing to discover relevant new information before the rest of the market

**please** do not confuse randomness in price changes with irrationality in price levels

→ predictability in stock prices implies market inefficiency because it indicates that investors are not using all available information

**efficient market hypothesis:** security prices fully (and correctly) reflect all available information, and hence it is impossible to earn economic profits by trading on that information
versions of the efficient market hypothesis 1.5

classic definition of market efficiency is relative to the information set

**weak**: market trading historical data
  example: past prices, trading volume, short interest

**semi-strong**: market trading historical data + prospects of the firm
  example: balance sheet composition, earning forecasts

**strong**: all relevant information, including insider information
  example: SEC restrictions on trading by corporate officers
if stock prices respond sluggishly to fundamental supply and demand factors, it makes sense to search for and exploit predictable patterns in stock prices → chartist vs econometrician


EMH implication: technical analysis is completely useless!!!

empirical evidence: earlier results show that some technically oriented trading strategies would have generated abnormal profits in the past, though...reality check says luck → nonrecurrent gains
Fundamental analysis

Research on the determinants of the present discounted value of all the payments a stockholder will receive from each share of stock.

Example: earnings and dividends prospects
- risk profile of the firm
- expectations for future interest rates

**EMH implication:** it will add value only if better than the others’

**Tools:**
- Macroeconomic and industry analysis [BMK05, chapter 17]
- Equity valuation [BMK05, chapter 18]
- Financial statement analysis [BMK05, chapter 19]
Testing weak efficiency

**statistical tools:** serial correlation vs serial dependence
filter rules vs reality check

**empirical results:** short vs long horizons \(\rightarrow\) dividends-price ratio
momentum vs contrarian strategy

**issues:** market microstructure effects

some funds seem to perform persistently well, even after controlling
for risk through market betas \(\rightarrow\) **skill?** not necessarily, multifactor
performance attribution models show that most of these funds follow
fairly mechanical styles rather than persistent skill at stock selection
Predictability: martingale vs random walk

**martingale**

\[ \mathbb{E}_t(P_{t+1}) \equiv \mathbb{E}(P_{t+1} | \mathcal{F}_t) = P_t \ \Rightarrow \ \mathbb{E}_t(r_{t+1}) = 0 \]

**random walk**

\[ P_{t+1} = \delta_{t+1} + P_t + \epsilon_{t+1} \]
\[ \delta_{t+1} \sim \text{deterministic component} \]
\[ \epsilon_{t+1} \sim \text{white noise component} \]

**remark**

as for unit root tests, interest lies on persistence rather than on predictability and hence one consider the more general setup \( \epsilon_{t+1} \sim I(0) \) process
Testing semi-strong efficiency

relevance: fundamental analysis uses a much richer information set to create portfolios than technical analysis

evidence: surprisingly, several easily accessible statistics seem to help predict abnormal risk-adjusted returns \(\rightarrow\) market anomalies

example: price-earning ratio and dividend yield
  sluggish response to firms’ earning announcements
  market capitalization \(\rightarrow\) small-firm (in-January) effect
  book-to-market ratio \(\rightarrow\) positive effect due to optimism?

joint-hypothesis issue: how to adjust for risk?
  CAPM vs multifactor interpretation
Competition as a source of efficiency

**incentive constraint:** investors will spend time and resources to gather and process new information only if such activity is likely to generate higher investment returns


→ pricing efficiency may differ across markets and securities

  example: emerging markets, small versus large stocks

competition among the many well-backed, highly paid, aggressive security analysts ensures that, as a general rule, stock prices ought to reflect available information regarding their proper levels
competition among investors ensures that only serious analyses and uncommon techniques are likely to entail the differential insight that may generate trading profits

e.g.: serious → financial economics, econometrics
      uncommon → behavioral finance vs chartism, econophysics

security analysis is economically feasible only for managers of large portfolios, hence the small investor is better off pooling resources in a mutual fund so as to obtain the advantages of large size

portfolio: index or not to index?
Role for rational portfolio management

→ efficient diversification requires competent security analysis

→ rather than beating the market, portfolios must match investors’ profiles, such as, for example, age, tax bracket, risk aversion, and employment

examples: (1) investors holding executive stock options

(2) older investors who are essentially living off savings must avoid long-term bonds, whose market values fluctuate dramatically with interest rates changes
main investment implication of the efficient market hypothesis is that profit opportunities to better-informed traders are at the expense of less-informed traders, but... there are other implications as well!!!

deviations from informational efficiency would also result in a large cost to the overall economy → inefficient resource allocation

example: if the value of biotech assets as reflected in the stock prices of biotech firms exceed the cost of acquiring those assets, the managers of such firms will have a strong signal that the market will regard further investments in the firm as a venture with positive net present value
Modern definition of market efficiency

unpredictability is equivalent to market efficiency only under a very special case → **constant risk premium** + **frictionless economy**

**general case:** financial markets are predictable to some degree, but far from being a symptom of inefficiency or irrationality, predictability is oil that lubricates the gears of capitalism

classic efficient market hypothesis ignores one of the central insights of modern financial economics → **risk-return tradeoff**

**modern notion:** profit as economic rent (competitive advantage) control for risk premium
Efficient market hypothesis under scrutiny

professional portfolio managers are among the first to cast a stone for obvious reasons, but... the debate will probably never end for three empirical issues

magnitude: relative contribution with respect to market volatility

joint hypothesis: every test of market efficiency depends on risk-premium assumptions

lucky event: for any fixed period of time, there exists at least one winning investment scheme
Fad hypothesis

if stock prices overreact to relevant news, they exhibit positive serial correlation (i.e., momentum) over short time horizons, but negative serial correlation over longer horizons due to subsequent correction → excess volatility due to the overshooting

spurious evidence?

(1) instead of the stock market fad interpretation, one could think of mean reversion in stock prices as a rational response of market prices to time-varying market risk premia (i.e., discount rate)

(2) building a long series of returns over a long horizon requires time span → structural breaks vs small sample
Predictors of broad market movements

there are several papers that aim at identifying observed variables that are able to predict market returns

definition: return on the aggregate stock market tends to be higher when the dividend yield (i.e., D/P ratio) is high

spurious evidence? these variables are probably proxying for the variation in the market risk premium

definition: given a dividend level, the stock price will be lower and the dividend yields will be higher when the risk premium (and hence the expected market return) is larger
Behavioral interpretation

**forecasting errors:** excessive weight to recent experience relative to prior beliefs typically entail P/E and book-to-market effects due to optimism

**overconfidence:** (1) dominance of active portfolio management in the face of the typical underperformance of such strategies, (2) men trade far more actively than women, though high trading activity is highly predictive of poor investment performance

**regret avoidance:** preference for conventional decisions accords with size and book-to-value effects
So, are markets efficient?

there are enough anomalies in the empirical evidence to justify the search for mispriced securities, though... one must look with many grains of salt at supposedly superior trading strategies

financial markets are near efficient and competitive enough to ensure that there is no free lunch (or easy picking)

Scheinkman & Xiong, 2005
Heterogeneous beliefs, speculation and trading in financial markets
(http://www.princeton.edu/~joses/wp/survey.pdf)
Empirical Finance

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Lecture 02: Predictability

references
Lo & MacKinlay, chapters 2 and 5
Cuthbertson & Nitzsche, chapter 4
Campbell, Lo & MacKinlay, chapter 2
Why do we care about predictability?

→ classic definition of market efficiency
   (or shall I say, the classic confusion?)

→ profitable quantitative trading strategies
   (even academics wish to beat the market!)

→ better understanding of the financial markets
   (after all, we must know what we are dealing with!)

→ never-ending debate about technical analysis
   (if we could just translate their jargon...)
Does the latter results from linguistic barriers? 2.2

technical analysis  The evident presence of support and resistance levels, coupled with a one-third retracement parameter when prices lie between them, suggests the presence of strong buying and selling opportunities in the near term.

time-series econometrics  The magnitudes and decay pattern of the first twelve sample autocorrelations and the statistical significance of the portmanteau Q-statistic suggest the presence of a high-frequency predictable component in stock returns.

→ past prices contain information for predicting future returns, though...  most readers find one statement plausible, whereas the other puzzling or, even worse, offensive.
Main stylized facts

literature on the predictability in asset returns is **too vast** and hence it is impossible to provide a complete survey in just a few pages/hours

**textbook survey**
- weekly and monthly data
- sample from 1962 to 1994

**empirical analysis**
- daily and monthly data
- sample from 1980 to 2005

**stylized facts**
- individual stocks vs stock market indexes
- daily frequency vs monthly frequency
- evolution over time
## Textbook survey

### Index sample first-order autocorrelations

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### Textbook survey

#### Weekly sample first-order autocorrelations

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Textbook survey
Weekly sample cross-autocorrelations

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Textbook survey
Weekly sample cross-autocorrelations

returns on size-sorted portfolios → strong cross-correlation

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\end{align*}
returns on size-sorted portfolios $\rightarrow$ asymmetric lead-lag pattern

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How to profit from cross-autocorrelations?

**Contrarian strategy:** portfolio with long positions on losers and short positions on winners

**Profitability:** mean reversion due to overreaction and to thin trading positive cross-autocorrelation due to thin trading

**Empirical and numerical evidence**

$\rightarrow$ cross-autocorrelation is much more relevant than overreaction

$\rightarrow$ market microstructures appeals more than fad hypothesis

**Caveat:** focus on profitability without controlling for risk!!
Investment horizon

**short horizon** predictability is strong and consistent through time

→ returns typically display mean reversion as well as cross-autocorrelations

**long horizon** mixed evidence of predictability

→ difficult problem of statistical inference

multiple time horizons, sensitive to sample period

to the extent that **transaction costs** are greater for trading strategies exploiting short-run horizon predictability, long-horizon predictability is perhaps a **more genuine** form of profit opportunity, **though**... one must account for stop-loss procedures and liquidity squeezes
Empirical Finance

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Lecture 03: Market Microstructures

references
Lo & MacKinlay, chapters 4 and 5
Campbell, Lo & MacKinlay, chapter 3
Usual assumptions

3.1

continuous unique prices vs multiple prices in ticks
evenly spaced in time vs transaction data
market clearing vs market makers

prices must fully reflect the available information in the market in the long run, though... market liquidity, competition/collusion among market makers, and asymmetric information are of major importance especially in the short run

main topics
nonsynchronous trading
price discreteness
bid-ask spread
As suggested in class, we plotted the distribution of the cents by year and by tenth. From the first graph we can observe that zero cents is the most frequent occurrence followed by 5 cents. We cannot see any pattern on the second graph.
Nonsynchronous trading: Example

assets  
   A  →  lots of liquidity
   B  →  thin trading

common shock  near market close  →  end-of-the-day price of asset A is more likely to reflect this information than B’s, though... price of asset B will eventually catch up

spurious effects  cross-autocorrelation  →  lead-lag pattern
               mean reversion in asset B
Statistical model of nontrading

Lo & McKinlay (Journal of Econometrics, 1990) effects of nontrading as a purely statistical artifact

unobserved continuously compounded return $r_{it} \rightarrow$ virtual returns representing changes in the underlying value of the security in the absence of trading frictions and institutional rigidities

**assumption:** fixed nontrading probability $\pi_i$

(independent of $r_{i,t} \rightarrow$ no economic motivation)

$\rightarrow$ nontrading process forms an iid sequence of ‘coin tosses’, with different nontrading probabilities across securities
Notation

nontrading indicator

$$\delta_{it} = \begin{cases} 
1 & \text{with probability } \pi_i \\
0 & \text{with probability } 1 - \pi_i 
\end{cases}$$

$k$-consecutive nontrading indicator

$$X_{it}(k) \equiv (1 - \delta_{it})\delta_{it-1} \ldots \delta_{it-k} = \begin{cases} 
1 & \text{with probability } (1 - \pi_i)\pi_i^k \\
0 & \text{with probability } 1 - (1 - \pi_i)\pi_i^k 
\end{cases}.$$ 

assumptions

$$\delta_{it} \perp \delta_{jt} \text{ for every } i \neq j$$

$$\delta_{it} \sim \text{iid over time}$$
What sort of returns does one observe then?

\[ r_{it}^* = \sum_{k=0}^{\infty} X_{it}(k)r_{it-k} \]

\[
\begin{align*}
0 & \quad \text{with probability } \pi_i \\
r_{it} & \quad \text{with probability } (1 - \pi_i)^2 \\
r_{it} + r_{it-1} & \quad \text{with probability } (1 - \pi_i)^2 \pi_i \\
\vdots & \quad \vdots \\
r_{it} + \ldots + r_{it-k} & \quad \text{with probability } (1 - \pi_i)^2 \pi_i^k \\
\vdots & \quad \vdots \\
\end{align*}
\]

\text{observed return} \quad \text{random sum of all past virtual returns} \quad \text{spurious autocorrelation arises}
Individual security returns

virtual returns dynamics
\[ r_{it} = \mu_i + \beta_i f_t + v_{it}, \quad i = 1, \ldots, N \]

assumptions
\[ f_t \sim \text{iid} \rightarrow \text{autocorrelation due only to nontrading} \]
\[ f_t \perp v_{it-k} \text{ for every } i, t, \text{ and } k \]

no mean effect
\[ \rightarrow \mathbb{E}(r^*_t) = \mu_i \]

second-order effects
\[ \rightarrow \text{var}(r^*_t) = \sigma_i^2 + \frac{2\pi_i}{1-\pi_i} \mu_i^2 \]

\[ \text{cov}(r^*_it, r^*_jt+k) = \begin{cases} -\mu_i^2 \pi_i^k & \text{for } i = j, k > 0 \\ \frac{(1-\pi_i)(1-\pi_j)}{1-\pi_i \pi_j} \beta_i \beta_j \sigma_f^2 \pi_j^k & \text{for } i \neq j, k \geq 0 \end{cases} \]
Individual security returns

virtual returns dynamics \[ r_{it} = \mu_i + \beta_i f_t + v_{it}, \quad i = 1, \ldots, N \]

assumptions \[ f_t \sim \text{iid} \implies \text{autocorrelation due only to nontrading} \]
\[ f_t \perp v_{it-k} \text{ for every } i, t, \text{ and } k \]

no mean effect \[ \implies \mathbb{E}(r^*_it) = \mu_i \]

second-order effects \[ \implies \text{var}(r^*_it) = \sigma_i^2 + \frac{2\pi_i}{1-\pi} \mu_i^2 \]

\[ \text{cov}(r^*_it, r^*_{jt+k}) = \begin{cases} 
-\mu_i^2 \pi_i^k & \text{for } i = j, k > 0 \\
(1-\pi_i)(1-\pi_j) \beta_i \beta_j \sigma_f^2 \pi_i^k & \text{for } i \neq j, k \geq 0 
\end{cases} \]
Individual security returns

virtual returns dynamics \[ r_{it} = \mu_i + \beta_i f_t + \upsilon_{it}, \quad i = 1, \ldots, N \]

assumptions \[ f_t \sim \text{iid} \rightarrow \text{autocorrelation due only to nontrading} \]
\[ f_t \perp \perp \upsilon_{it-k} \text{ for every } i, t, \text{ and } k \]

no mean effect \[ \rightarrow \mathbb{E}(r^*_{it}) = \mu_i \]
second-order effects \[ \rightarrow \operatorname{var}(r^*_{it}) = \sigma_i^2 + \frac{2\pi_i}{1-\pi_i} \mu_i^2 \]

\[
\operatorname{cov}(r^*_{it}, r^*_{jt+k}) = \begin{cases} 
-\mu_i^2 \pi_i^k & \text{for } i = j, k > 0 \\
\frac{(1-\pi_i)(1-\pi_j)}{1-\pi_i\pi_j} \beta_i \beta_j \sigma_f^2 \pi_j^k & \text{for } i \neq j, k \geq 0
\end{cases}
\]
virtual returns dynamics

\[ r_{it} = \mu_i + \beta_i f_t + v_{it}, \quad i = 1, \ldots, N \]

assumptions

\[ f_t \sim \text{iid} \rightarrow \text{autocorrelation due only to nontrading} \]
\[ f_t \perp v_{it-k} \text{ for every } i, t, \text{ and } k \]

no mean effect

\[ \mathbb{E} (r^*_{it}) = \mu_i \]

second-order effects

\[ \text{var} (r^*_{it}) = \sigma^2_i + \frac{2\pi_i}{1-\pi_i} \mu^2_i \]

\[ \text{cov} (r^*_{it}, r^*_{jt+k}) = \begin{cases} -\mu^2_i \pi^k_i & \text{for } i = j, k > 0 \\ \frac{(1-\pi_i)(1-\pi_j)}{1-\pi_i \pi_j} \beta_i \beta_j \sigma^2_f \pi^k_j & \text{for } i \neq j, k \geq 0 \end{cases} \]
Individual security returns

virtual returns dynamics

\[ r_{it} = \mu_i + \beta_i f_t + \nu_{it}, \quad i = 1, \ldots, N \]

assumptions

\[ f_t \sim \text{iid} \quad \longrightarrow \quad \text{autocorrelation due only to nontrading} \]
\[ f_t \perp \perp v_{it-k} \quad \text{for every } i, t, \text{ and } k \]

no mean effect

\[ \mathbb{E}(r^*_{it}) = \mu_i \quad \longrightarrow \quad \text{asymmetric} \]

second-order effects

\[ \text{var}(r^*_{it}) = \sigma_i^2 + \frac{2\pi_i^2}{1-\pi_i} \mu_i^2 \quad \longrightarrow \quad \text{lead-lag pattern} \]

\[ \text{cov}(r^*_{it}, r^*_{jt+k}) = \begin{cases} 
-\mu_i^2 \pi_i^k & \text{for } i = j, k > 0 \\
\frac{(1-\pi_i)(1-\pi_j)}{1-\pi_i\pi_j} \beta_i \beta_j \sigma_f^2 \pi_j^k & \text{for } i \neq j, k \geq 0
\end{cases} \]
assumption group securities by their nontrading probabilities and then form equal(-geometrically)-weighted portfolio

diversification argument \[ r_{pt}^* = \mu_p + (1 - \pi_p) \bar{\beta}_p \sum_{k=0}^{\infty} \pi_p^k f_{t-k} \]

\[
\text{cov} \left( r_{pt}^*, r_{qt+k}^* \right) = \begin{cases} 
\frac{1-\pi_p}{1+\pi_p} \bar{\beta}_p^2 \sigma_f^2 \pi_p^k & \text{for } p = q, k \geq 0 \\
\frac{(1-\pi_p)(1-\pi_q)}{1-\pi_p\pi_q} \bar{\beta}_p \bar{\beta}_q \sigma_f^2 \pi_q^k & \text{for } i \neq j, k \geq 0
\end{cases}
\]

conclusion AR(1) with coefficient equal to nontrading probability asymmetric lead-lag patterns
Further results

**time aggregation**

→ spurious autocorrelation remains negative for individual securities, though it declines monotonically in magnitude as the sampling interval increases

→ expected returns aggregate linearly, whilst variances do not due to the presence of negative serial correlation
  (variance of the sum < sum of the variances)

**calibration results**

impact of nontrading is negligible in the short horizon for individual stocks, but substantial for portfolios
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Lecture 04: Asymmetric Information

reference
Adverse selection costs

**bid-ask spread**: execution and order processing costs $\rightarrow$ constant?
monopoly power $\rightarrow$ competition?
inventory costs $\rightarrow$ risk aversion
adverse selection costs $\rightarrow$ endogenous!!!

**Easley & O’Hara**: sequential trading model that individuates
the effects of asymmetric information

**insider vs uninformed**
market maker is unable to identify who is who, hence...
bid-ask spread $\rightarrow$ reward to provide liquidity under
asymmetric information
Assumptions

**single market maker:** risk neutral, competitive, updates by Bayes

asset value $V$ with signal $\psi \in \{L, 0, H\}$

\[
\begin{align*}
\mathbb{E}(V | \psi = L) &= V_L \\
\mathbb{E}(V | \psi = H) &= V_H \\
\mathbb{E}(V | \psi = 0) &= \delta V_L + (1 - \delta)V_H = V^* 
\end{align*}
\]

**informed traders:** risk neutral, always trade

**uninformed traders:** liquidity and portfolio reasons
information event with probability $\alpha \rightarrow$ Dow-Jones rumor wire

if there are news, then
- bad ($\psi = L$) with probability $\delta$
- good ($\psi = H$) with probability $1 - \delta$

trader population: insider with probability $\mu$
- uninformed with probability $1 - \mu$

uninformed trader
- potential seller with probability $\gamma \rightarrow$ trades with probability $\epsilon_S$
- potential buyer with probability $1 - \gamma \rightarrow$ trades with probability $\epsilon_B$
Tree diagram of the trading process

---

- **nature plays once per day**

- **sequential trading game**
market maker always assign a nonzero probability for the good and bad states of the nature, hence the bid and ask prices will always lie between $V_L$ and $V_H$ → insider always trade

no-trade event carries as much information as buys and sells, because it is more likely to occur when there are no news

volume signals whether there is new information
inventory position (or signed volume) signals its direction

preliminary conclusions
→ it takes volume to move prices
→ quote revision depends on the number of buys, sells and no-trades
Evolution of beliefs

\((n_t, \beta_t, s_t)\) is a sufficient statistic for trade history \(Q^t\)

\[ n_t \text{ number of no-trade outcomes} \]
\[ \beta_t \text{ number of buys} \]
\[ s_t \text{ number of sells} \]

**example:** probability MM assigns to the absence of new information

\[
p_{0t} = \Pr(\psi = 0 \mid Q^t) \\
= (1 - \alpha)(\gamma \epsilon_s)^{s_t}[(1 - \gamma)\epsilon_B]^\beta_t \left\{ (1 - \alpha)(\gamma \epsilon_S)^{s_t}[(1 - \gamma)\epsilon_B]^\beta_t \\
+ (1 - \mu)^{n_t} [\alpha \delta(\mu + (1 - \mu)\gamma \epsilon_S)^{s_t}((1 - \mu)(1 - \gamma)\epsilon_B)^\beta_t \\
+ \alpha(1 - \delta)((1 - \mu)\gamma \epsilon_S)^{s_t}(\mu + (1 - \mu)(1 - \gamma)\epsilon_B)^{\beta_t}] \right\}^{-1}.
\]
the evolution of beliefs will completely determine the evolution of quotes and, as the market maker is competitive, the bid/ask quotes will coincide with the expected value of the stock given that the next trade outcome is a sell/buy, respectively

\[
b_{t+1} = \mathbb{E}(V \mid Q_t, Q_{t+1} = S) \\
= V_L \Pr(\psi = L \mid Q_t, S) + V_* \Pr(\psi = 0 \mid Q_t, S) + V_H \Pr(\psi = H \mid Q_t, S)
\]

\[
a_{t+1} = \mathbb{E}(V \mid Q_t, Q_{t+1} = B) \\
= V_L \Pr(\psi = L \mid Q_t, B) + V_* \Pr(\psi = 0 \mid Q_t, B) + V_H \Pr(\psi = H \mid Q_t, B)
\]

**transaction price:** optional sampling yields a subordinated process
Implications

1. quotes change even in the absence of trades
2. bid-ask spread decreases with the trade duration
3. trade durations are endogenous and serially dependent
4. prices do not satisfy the Markov property
5. transaction prices are martingales, hence weak efficiency
6. strong efficiency holds only in the limit
7. positive correlation between volatility and volume
Extensions

1. time-varying probability of uninformed trading
   nonspeculative reasons for uninformed trading may depend on the business cycle

2. size effect: block trades vs small trades
   insiders wish to profit as much and quick as possible, hence they may prefer block trades

3. option market
   options offer more leverage to the insiders, hence it may affect the partial equilibrium conclusions
it is straightforward to estimate all the parameters of the sequential trading model by Easley & O’Hara using transactions data so as to retrieve the unconditional probability of insider trading

\[ \text{PIN} = \frac{\alpha \mu}{\alpha \mu + \epsilon_S + \epsilon_B} \]

\[
\rightarrow \text{measure of adverse selection cost}
\]

the arbitrage pricing theory says that one must pay a premium for all sources of risk, hence it follows that expected returns must increase with PIN
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Lecture 05: CAPM, APT, and Multifactor Models

references
Campbell, Lo & MacKinlay, chapters 5 and 6
Cuthbertson & Nitzsche, chapter 8
Factor models in finance

5.1

linear factor decomposition not only entails arbitrarily good fit for a sufficient number of factors but also simplifies asset pricing

\[ \text{covariance as a measure of risk} \]

examples

**CAPM** \[ R_t = \alpha + \beta R_{Mt} + \epsilon_t \]

**APT** \[ R_t = a + B f_t + \epsilon_t \]

**ICAPM** \[ R_{it} = \alpha_{it} + \beta_{it}^M R_{Mt} + \beta_{it}^{SMB} R_{SMBt} + \beta_{it}^{HML} R_{HMLt} + \beta_{it}^{MOM} R_{MOMt} + \beta_{it}^{PIN} R_{PINt} + \epsilon_{it} \]
CAPM: Time-series aspects

\[ R_t = \alpha + \beta R_{Mt} + \epsilon_t \]  \hspace{1cm} \text{(what sort of returns are we talking about?)}

**testing implications**
- excess returns version \( \rightarrow \alpha = 0 \)
- zero-beta real returns \( \rightarrow \alpha = (\iota - \beta) \gamma \)

**remarks**
one must restrict the cross-sectional dimension to no larger than about ten assets, whereas increase the length of the time series as much as possible

**evidence**
CAPM pricing error is twofold the conditional CAPM pricing error where the market beta depends on the business cycle (e.g., term premium and price-earning ratio) for size-sorted portfolios
CAPM: Cross-section aspects

\[ R_{it} - R_{0t} = \beta_i (R_{Mt} - R_{0t}) + \epsilon_{it} \]  
\[ \bar{R}_i - \bar{R}_0 = \beta_i (\bar{R}_M - \bar{R}_0) + \bar{\epsilon}_i \]

(imposing CAPM restriction)  
(taking time averages)

cross-section beta regression:  
\[ \bar{R}_i = \lambda_0 + \lambda_1 \beta_i + v_i \]

testing implications  
market beta completely captures the cross-sectional variation of expected excess returns and market risk premium is positive  
\[ \lambda_0 = \bar{R}_0 \text{ and } \lambda_1 = \bar{R}_M - \bar{R}_0 > 0 \]

no omitted variables (e.g., idiosyncratic risk, BTM)

remarks  
individual stocks are too volatile to reject differences in expected excessive returns  
\[ \rightarrow \text{ sort stocks into portfolios} \]
errors-in-variables problem market beta is not observable and hence one must replace it by the time-series regression estimate

(1) OLS estimate of $\lambda_1$ is downward biased
(2) nonnormality may show up as an association between residual risk and returns
(3) association between true beta and idiosyncratic risk will show up as a significant idiosyncratic risk in the cross-section regression

remedy one may group stocks into portfolios sorted either by beta or market capitalization, otherwise instrumental approach
CAPM: Cross-section evidence

Data 10 size-sorted NYSE stock portfolios
government and corporate bond portfolio

Preliminary evidence
(1) bond portfolios have low beta and low average returns
(2) size-sorted stock returns are positively related to beta

Deeper results
(1) bond and stock returns lie respectively below and above the SML
(2) average return for smallest stock decile is well above SML

Conclusion CAPM explain stock returns relative to bond returns, but fails to elucidate small-firm effect → size anomaly
CAPM: Rolling regressions

Fama & MacBeth (JPE 1973) suggests undertaking a separate cross-section regression for each time period, hence obtaining a time series of regression coefficients on which one may perform tests

\[ R_t = \alpha'_t + \lambda_t \beta + \gamma_t Z_t + \epsilon_t \]

\( Z_t \sim \) additional cross-section characteristics

**CAPM implications**  \( \lambda_t > \alpha_t = \gamma_t = 0 \) for every \( t \)

**Fama-MacBeth approach**  tests on time-series averages

**errors-in-variables issue**  \( \rightarrow \) either instrumental variables or size-sorted portfolios
Roll’s critique by definition, market portfolio is mean-variance efficient and so the CAPM-implied SML must necessarily hold in the sample → empirical violations of the SML thus merely indicate that the market portfolio proxy is not adequate

recent trend multifactor models (e.g., APT and ICAPM) that are able to cope with the cross-sectional variation of expected returns

remark statistical factor models are a convenient way to reduce the dimensionality of the problem while keeping the explanatory power, though they often lack interpretability
→ linear combination of economic variables
→ number of significant factors increases with number of securities
Fama and French (JFE 1993) entertain three factors to explain the monthly returns on the 25 portfolios sorted by size and value.

**time series** \[ R_{it} = \beta_i^M R_{Mt} + \beta_i^{SMB} R_{SMBt} + \beta_i^{HML} R_{HMLt} + \epsilon_{it} \]

**cross-section** \[ \bar{R}_i = \lambda_M \beta_i^M + \lambda_{SMB} \beta_i^{SMB} + \lambda_{HML} \beta_i^{HML} + \bar{\epsilon}_i \]

CAPM implies perfect correlation between market beta and expected returns, **though...** points in the scatter plot are all over the place.

1. HML and SMB betas explain 90% of cross-sectional variation.
2. CAPM fails mainly because of BTM sorting (negative correlation).
3. Fama-French factor model fails to explain momentum sorting.
Size and Book-to-Market Ratio Quintile Portfolios

excess returns sorted by book-to-market ratio within a given size quintile

excess returns sorted by size within a given book-to-market ratio quintile
The two lines connect portfolios of different size categories, within a given book-to-market category. We only connect the points within the highest and lowest BMV categories. If we had joined up point for the other BMV quintiles, the lines would show a positive relationship, like that for the value stocks – showing that the predicted returns from the Fama-French 3 factor model broadly predict average returns on portfolios sorted by size and BMV.
Where do market anomalies come from?

size effect informational risk $\rightarrow$ PIN factor
  low price phenomenon $\rightarrow$ January effect

value effect typical value firms are in financial distress and hence
  low BTM may signal aggregate risk over business cycle

momentum effect fad hypothesis $\rightarrow$ overreaction
  business cycle story

macroeconomic factors (e.g., dividend-price ratio, term spread and
  consumption-wealth ratio) help predicting stock returns (but not their
  cross-sectional variation) and recessions, reinforcing the business-
  cycle justification behind the factor-mimicking portfolios
Cross-equation restrictions and the APT

APT assumes that $R_{it} = \mathbb{E}(R_{it}) + \sum_{j=1}^{k} b_{ij} f_{jt} + \epsilon_{it}$ with zero-mean factors, yielding

$$R_{it} = \lambda_0 + \sum_{j=1}^{k} \lambda_j b_{ij} + \sum_{j=1}^{k} b_{ij} f_{jt} + \epsilon_{it}$$

due to the beta decomposition given by $\mathbb{E}(R_{it}) = \lambda_0 + \sum_{j=1}^{k} \lambda_j b_{ij}$

**regression**

$$R_{it} - R_{0t} = \alpha_i + \sum_{j=1}^{k} b_{ij} f_{jt} + \epsilon_{it}$$

with $R_{0t}$ playing the role of $\lambda_0$ and $\alpha_i = \sum_{j=1}^{k} \lambda_j b_{ij}$

$\rightarrow$ estimation by NLSUR to account for the contemporaneous covariances between error terms
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Lecture 06: Fund Performance

reference Cuthbertson & Nitzsche, chapter 9
Performance measure: Alpha

Jensen
CAPM single-factor model $\rightarrow$ market portfolio

Fama-French
three-factor model $\rightarrow$ BTM ratio, size

Carhart
four-factor model $\rightarrow$ momentum

conditional
time-varying alphas and betas

$$R_{it} = \alpha_{it} + \beta_{it}^{M} R_{Mt} + \beta_{it}^{SMB} R_{SMBt} + \beta_{it}^{HML} R_{HMLt} + \beta_{it}^{MOM} R_{MOMt} + \epsilon_{it}$$
unconditional performance measures do not accommodate a scenario
in which fund managers identify changing market information about
expected returns and risk

\[ \alpha_{it} = \alpha_{0i} + A_i'Z_{t-1} \quad \beta_{it} = \beta_{0i} + B_i'Z_{t-1} \]

where \( Z_t \) is a vector of zero-mean instruments for the (macro)economic
information set \( \rightarrow \text{interaction} \) capturing the covariance between
time-varying instruments and risk factors

**examples**  
one-month T-bill yield, quality spread in the corporate
bond market, dividend yield, term structure slope
Mutual Funds: Evidence in UK

**Data** returns on bid-price-to-bid-price basis with reinvested gross dividends for an equal-weighted portfolio of 752 surviving and non-surviving funds from 1978 to 1997

**Risk adjustment** Fama-French three-factor model

**Result** robust evidence of a statistically significant negative alpha
  → style: growth, income, general equity, small caps
  → value-weighted portfolios
  → transaction costs
Performance persistence in UK

**Data**  equal-weighted portfolios based on decile rankings of the funds’ either three-factor alphas or raw returns

despite spread of 3.54% in the annual compound return between the best and worst fund, it is difficult to beat the market because the annual turnover of such a strategy is around 80%, with a bid-ask spread of 5%

**alphas**  positive, but insignificant, for top two portfolios
            negative and significant for bottom two/four portfolios

**Conclusion**  high turnover and only poor performance persists
Alternative benchmarking

Blake and Timmermann (EFR, 1998)

\[ R_{it} - r_t = \alpha_i + \beta_i^M (R_{Mt} - r_t) + \beta_i^{SC} (R_{SCt} - R_{Mt}) + \beta_i^B (R_{Bt} - r_t) + \epsilon_{it} \]

market portfolio + small caps + 5-year government bond
persistence among both top- and bottom-performing funds

Carhart (JF, 1997)

\[ R_t = \alpha + \beta^M R_{Mt} + \beta^{SMB} R_{SMBt} + \beta^{HML} R_{HMLt} + \beta^{MOM} R_{MOMt} + \epsilon_t \]
lucky momentum effect drives earlier findings on the performance of mutual funds given that it explains half of the spread between top and bottom decile returns \( \rightarrow \) accidentally holding winning stocks
Fund characteristics and performance

Data
equal-weighted portfolios based on decile rankings of US funds’ turnover (i.e., trading activity)

top turnover decile significantly (at 10%) outperforms the bottom decile by 4.3% per annum over the period ranging from 1973 to 1995

Performance attribution
investment style, stock-selection ability, and market-timing ability

Risk adjustment
Carhart’s alpha yields no significant difference
significant correlation between performance and future cash flow, especially if one employs Carhart’s alpha. Sophisticated investors?

Funds receiving new money significantly outperform funds losing money, but do not significantly beat the market.

**Momentum effect**
Money flows into funds with the best past performance but does not flow out of funds with the worst past performance.

**Contrarian effect**
Too much money inflow may harm future performance for the fund, becomes too bulky and pressure for liquid investments increase.
Mutual fund managers

relationship between performance and cross-sectional characteristics of fund managers

**examples**  
schooling  $\rightarrow$ SAT scores vs MBA degree  
management  $\rightarrow$ single vs team

**flow of funds**  $\rightarrow$ **agency problems within the fund industry**  
investors would like fund managers to maximize risk-adjusted expected returns, whereas fund managers wish to maximize investment inflow

**shape of the flow-performance relationship**  $\rightarrow$ incentives that fund managers face may alter their risk-taking behavior
Hedge funds

**completely different animal relative to mutual funds**

- short sell even cash
- leverage through derivatives
- administration and performance fees
- lockup and redemption periods
- minimum investment

→ do not necessarily entail higher risk-adjusted returns, but bring about gains in diversification for hedge fund returns do have low correlations with other securities

→ risk adjustment must control for nonlinear payoffs of derivatives
Stars: Sheer luck or stock-picking ability?

**techniques**  
false discovery rate  
reality check using bootstrap  
random-effect panel regression

**evidence**  
lots of really poor performers  
very few funds really outperform  
not much persistence over time

**puzzle**  
why are there so many funds doing active management?  
→ gross vs net returns  
→ overconfidence
Lecture 07: Event Study

references

Campbell, Lo and MacKinlay, chapter 4
Cuthbertson & Nitzsche, chapter 9
Goals of event study analyses

financial economists often wish to measure the effect of an economic event on the value of a firm → market (near) efficiency dictates that the impact will immediately affect market prices

**magic** direct effects may take many months or even years of observation, though indirect measures based on asset prices require a relatively short time period

**examples** stock splits
merger and acquisitions
issues of new debt or equity
change in the regulatory environment
earnings or macroeconomic news announcements
First step: Event definition

**event of interest**  clear definition of the problem

**event window**  identify the period over which there might be an impact in the security prices of the firms involved in the event

**example**  earnings announcements → one usually expand the event window to include not only the announcement day but also the day after so as to capture price effects which occur after trading hours, though one may also investigate **information leakage** by looking at pre-event returns
Second step: Selection criteria

it is paramount to carefully determine the selection criteria for the inclusion of a given firm in the study in order to ensure the absence of sample selection bias and the representativeness of the sample.

**examples** data availability such as listing on the NYSE or AMEX restrictions such as membership in a specific industry

**what to do?** summarize the most relevant data characteristics (e.g., market capitalization, industry representation, distribution of events through time), and note any potential biases that may arise due to the sample selection.
Third step: Abnormal returns

difference between actual return and normal return, where the latter is the expected return in the absence of the event

(1) constant-mean-return model assumes that the expected return of a given security is constant through time

(2) market model assumes a stable linear relation between the market and security returns

(3) market-adjusted returns stem from a restricted variant of the market model with zero alpha and unit beta (e.g., IPOs)
Fourth step: Estimation procedure

estimate the parameters of the abnormal returns model using a subset of the data \(\rightarrow\) estimation window

example one may estimate the market-model parameters using daily data over the 120 days prior to the event

main rule estimation window does not include the event window, otherwise you could well contaminate the normal performance model parameter estimates
after calculating abnormal returns using the parameter estimates, we must design the testing framework in order to draw overall inferences for the impact of the event of interest in the abnormal returns.

(1) how to define the **null hypothesis**?
  → event has no impact on the behavior of security returns, though one may prefer to restrict attention to mean effects

(2) how to **aggregate** the abnormal returns of individual firms?
  → aggregation is along two dimensions, through time and across securities
presentation of the empirical findings follows the formulation of the econometric design, and hence it requires presentation of diagnostic tests to ensure that the statistical assumptions do indeed hold for the sample under consideration.

**Example** in the context of small samples, one or two influential observations in the sample may heavily affect the results.

→ it is important to understand the problems of your sample if you wish to gauge the importance of your results.
Seventh step: Interpretation

ideally the empirical results will lead to insights about the mechanisms by which the event affects security prices, though one must carefully distinguish between competing explanations

**robustness checks** one must also understand how sensitive the empirical findings are to the sample selection, to the normal returns model, and to the estimation and event windows
Example: Earnings announcements I

**sample** daily stock returns for the 30 firms of the Dow Jones Industrial Index from January 1988 to December 1993

**event window** 20 pre-event days + event day + 20 post-event days

**estimation window** 250 trading days prior to the event window

**data**
- date of the announcement (source: datastream)
- actual earnings (source: compustat)
- expected earnings (source: I/B/E/S)

**news**
- good if actual − expected > 2.5% (189 announcements)
- neutral if |actual − expected| < 2.5% (173 announcements)
- bad if actual − expected < −2.5% (238 announcements)
**Example: Earnings announcements II**

**constant-mean-return model** \( R_{it} = \mu_i + \xi_{it} \) with \( \xi_{it} \sim \mathcal{N}(0, \sigma_i^2) \) 
\rightarrow despite its simplicity, it often yields very similar results to those of more sophisticated models because the variance of the abnormal return does not vary much across models

**market model** \( R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \) with \( \epsilon_{it} \sim \mathcal{N}(0, \sigma_i^2) \) 
\rightarrow it removes the portion of the return that relates to the variation in the market portfolio return, reducing (though usually only slightly) the variance of the abnormal return

**market-adjusted-return model** \( R_{it} = R_{mt} + \epsilon_{it} \) with \( \epsilon_{it} \sim \mathcal{N}(0, \sigma_i^2) \) 
\rightarrow last resort in view of data unavailability, e.g., analysis of the underpricing of initial public offerings
**Example: Earnings announcements III**

**time line**
- estimation window from $T_0 + 1$ to $T_1$ \( \tau \in [-270, -21] \)
- pre-event window from $T_1 + 1$ to $T_2$ \( \tau \in [-20, -1] \)
- event window from $T_2 + 1$ to $T_3$ \( \tau = 0 \)
- post-event window from $T_3 + 1$ to $T_4$ \( \tau \in [1, 20] \)

**main assumption** event is **exogenous** with respect to the change in market value of the security, so that we may interpret the abnormal returns as a measure of the event impact on the value of the firm

**estimation:** OLS is consistent
- \((T_1 - T_0) \times 1\) vector from estimation window: \( \hat{\epsilon}_{i\tau} = R_{i\tau} - \hat{\alpha}_i - \hat{\beta}_i R_{m\tau} \)
- \((T_4 - T_1) \times 1\) vector from event window: \( \hat{\epsilon}^*_{i\tau} = R_{i\tau} - \hat{\alpha}_i - \hat{\beta}_i R_{m\tau} \)
Example: Earnings announcements IV

\[ \tilde{\epsilon}_i^* \sim \mathcal{N} \left( 0, \sigma_i^2 I_{T_4-T_1} + \sigma_i^2 P_{[\iota \ R^*_m]} \right) \]

**mean** zero meaning that the abnormal return forecasts are **unbiased**

**covariance matrix** involves the variance due to future disturbances and the additional variance due to **sampling error**

common sampling error leads to **serial correlation** of the abnormal returns unless the length of the estimation window \((T_1 - T_0)\) grows without bounds
Example: Earnings announcements V

aggregation cumulative abnormal returns $\hat{\text{CAR}}_i(\tau_1, \tau_2) = \gamma' \hat{\epsilon}_i^*$, where $\gamma$ is a $(T_4 - T_1) \times 1$ vector with ones from $\tau_1 - T_1$ to $\tau_2 - T_1$ and zeroes otherwise

individual test $\hat{\text{SCAR}}_i(\tau_1, \tau_2) = \hat{\text{CAR}}_i(\tau_1, \tau_2) / \hat{\sigma}_i(\tau_1, \tau_2) \sim t_{T_1 - T_0 - 2}$

aggregation under the assumption of no correlation across the abnormal returns of different securities

$$\overline{\text{SCAR}}(\tau_1, \tau_2) = \overline{\text{CAR}}(\tau_1, \tau_2) / \overline{\sigma}(\tau_1, \tau_2) \sim N(0, 1)$$

$$\overline{\text{CAR}}(\tau_1, \tau_2) = \gamma' \frac{1}{N} \sum_{i=1}^{N} \hat{\epsilon}_i^* \quad \overline{\sigma}^2(\tau_1, \tau_2) = \frac{1}{N^2} \sum_{i=1}^{N} \hat{\sigma}_i^2(\tau_1, \tau_2)$$
Example: Earnings announcements VI

Evidence strongly support that earnings announcements do indeed convey information useful for the valuation of firms.

**Good news**  sample average abnormal return of 0.965%  
with standard error of 0.104%  → **strong rejection**

**Neutral news**  sample average abnormal return of -0.091%  
with standard error of 0.098%  → **no rejection**

**Bad news**  sample average abnormal return of -0.679%  
with standard error of 0.098%  → **strong rejection**
assumption of no correlation is **reasonable** only if the event windows of the securities in the sample do not overlap in calendar time, which clearly is **not** the case in the earnings announcements example

**solutions**

1. apply the security level analysis to a portfolio that aggregates abnormal returns using event-time dating
   → allows for cross-correlation of the abnormal returns
2. examine abnormal returns without doing any aggregation by means of a multivariate regression model with dummy variables for the event date
   → accommodates partial clustering and asymmetric event effects, though poor finite-sample properties
Further issues and extensions

(1) one may allow for asymmetric event effects by regressing the cumulative abnormal returns on characteristics specific to the event observation (controlling for selection bias, if any)

(2) if the event timing is precise, then it always pays to go for shorter sampling intervals

(3) if the event timing is uncertain, it is much more convenient to expand the event window to two or more days than to rely on Bayesian methods

(4) bias due to thin trading does not play a major role, especially if one adjusts the market beta accordingly
Market efficiency in real time

Busse and Green (JFE 2002)  CNBC’s Morning Call and Midday Call segments broadcast during the market opening times analysts’ views about individual stocks, and hence provide a unique opportunity to study market efficiency

http://www.bus.emory.edu/cgreen/docs/cnbc/cnbc.html

Background information

event definition: ticker symbol appears on screen
trade intensity: 29 trades from -15 minutes to time 0
  29 trades in the first minute
volume: $1.7 million traded from time -15 minutes to 0
  $2.4 million in the first minute
Lecture 08: Present-value models

references
Campbell, Lo and MacKinlay, chapter 7
Cuthbertson & Nitzsche, chapter 10
focus of empirical asset pricing is weirdly on returns rather than prices,  
*though*... there is a strand of the literature that attempts to bring  
attention back to price behavior  

*present-value models* relate the price of a security to its expected  
future cash flows discounted to the present using either a constant  
or time-varying discount rate  

*examples*  
bonds $\rightarrow$ coupon payments  
stocks $\rightarrow$ dividends
dividends in all future periods enter the present-value formula and so the dividend in any one period is only a small component of the price discount rate between any one period and the next is only a small component of the long-horizon discount rate that one must apply to a distant future cash flow

**conclusions**
(1) long-lasting movements in dividends thus have much larger effects on prices than temporary movements do
(2) valuation models establish links between movements in prices, dividends, and returns, individuating the close connection between asset prices and long-horizon returns
return definition \( R_{t+1} \equiv \frac{P_{t+1} + D_{t+1} - P_t}{P_t} \) or \( r_{t+1} \equiv \log(1+R_{t+1}) \)

notation \( R_{t+1} \) denotes a holding-period return from \( t \) to \( t + 1 \)
\( P_t \) is the stock price at the end of time \( t \) (ex-dividend)

ex-dividend price purchase of a stock share at price \( P_t \) today gives one a claim to next period’s dividend per share \( D_{t+1} \) but not to this period’s dividend \( D_t \)
Linear present-value model

**Preliminary Assumption**  Constant expected return

\[
\mathbb{E}_t R_{t+1} = \mathbb{E}_t \left( \frac{P_{t+1} + D_{t+1} - P_t}{P_t} \right)
\]

\[
R = \frac{1}{P_t} \mathbb{E}_t \left( P_{t+1} + D_{t+1} \right) - 1 \Rightarrow P_t = \mathbb{E}_t \left( \frac{P_{t+1} + D_{t+1}}{1 + R} \right)
\]

Solve difference equation forward \( \kappa \) periods by substituting out future prices and using the law of iterated expectations

\[
P_t = \mathbb{E}_t \left[ \sum_{\ell=1}^{\kappa} (1 + R)^{-\ell} D_{t+\ell} \right] + \mathbb{E}_t \left[ (1 + R)^{-\kappa} P_{t+\kappa} \right]
\]

↓ ↓ ↓ ↓  dividends present value  ↓ ↓ ↓  bubble
Absence of bubbles

unless you expect prices to grow forever at rate $R$ or faster, the bubble component must shrink to zero as time passes

$$\lim_{\kappa \to \infty} \mathbb{E}_t \left[ (1 + R)^{-\kappa} P_{t+\kappa} \right] = 0$$

though... models of rational bubbles relax this assumption

implication taking limits yields the stock price as the expected present value of future dividends out to infinite future, discounted at a constant rate

$$P_t = \mathbb{E}_t \left[ \sum_{\ell=1}^{\infty} (1 + R)^{-\ell} D_{t+\ell} \right]$$
unrealistic special case that nevertheless provides some useful intuition occurs when dividends are also expected to grow at a constant rate $G < R$ (otherwise stock price diverges)

$$P_t = \mathbb{E}_t \left[ \sum_{\ell=1}^{\infty} (1 + R)^{-\ell} D_{t+\ell} \right] = \sum_{\ell=1}^{\infty} (1 + R)^{-\ell} \mathbb{E}_t D_{t+\ell}$$

$$= \sum_{\ell=1}^{\infty} (1 + R)^{-\ell} (1 + G) \mathbb{E}_t D_{t+\ell-1} = \sum_{\ell=1}^{\infty} (1 + R)^{-\ell} (1 + G)^{\ell} \mathbb{E}_t D_{t}$$

$$\sum_{\ell=1}^{\infty} (1 + R)^{-\ell} (1 + G)^{\ell} D_t = \frac{1 + G}{R - G} D_t$$
Remarks on Gordon growth model

(1) elasticity of price with respect to discount rate is \(-\frac{R}{R - G}\), meaning that stock price is extremely sensitive to permanent changes in the discount rate \(R\) if the latter is close to \(G\).

(2) no assumptions on equity repurchases by firms, which would affect the time pattern of expected future dividends per share, but not the validity of the present-value formula.

(3) assuming a constant expected return through time is known as the martingale model of stock prices, despite that fact that \(\mathbb{E}_t P_{t+1} = (1 + R)P_t - \mathbb{E}_t D_{t+1} \neq P_t\).

\(\longrightarrow\) to obtain a martingale, one must construct a portfolio that reinvests every dividend payment in the stock.
constant expected returns imply that stock prices will cointegrate with dividends if they are both nonstationary → present-value formula yields a stationary linear combination of prices and dividends

\[
P_t - \frac{1}{R} D_t = \sum_{\ell=1}^{\infty} \left( \frac{1}{1+R} \right)^\ell \mathbb{E}_t D_{t+\ell} - \frac{1}{R} D_t
\]

\[
= \sum_{\ell=1}^{\infty} \left( \frac{1}{1+R} \right)^\ell \mathbb{E}_t \left[ D_{t+\ell} - D_t \right]
\]

\[
= \sum_{\ell=1}^{\infty} \left( \frac{1}{1+R} \right)^\ell \mathbb{E}_t \left[ \Delta D_{t+\ell} + \ldots + \Delta D_{t+1} \right]
\]

\[
= \frac{1}{1+R} \mathbb{E}_t \Delta D_{t+1} + \left( \frac{1}{1+R} \right)^2 \mathbb{E}_t \left[ \Delta D_{t+1} + \Delta D_{t+2} \right] + \ldots
\]

\[
= \frac{1}{R} \sum_{\ell=0}^{\infty} (1 + R)^{-\ell} \mathbb{E}_t \Delta D_{t+\ell+1}
\]

↓   ↓
if expected dividend growth converge, then stationary
Time-varying expected returns

albeit the analytical convenience, assuming constant expected returns is not consistent with predictability → under time-varying expected returns, present-value relations become nonlinear

solution loglinear approximation of the relation between prices, dividends, and returns

accounting sanity check high prices indicate some combination of high (expected) future dividends and low (expected) future returns

starting point

\[ r_{t+1} = \log (P_{t+1} + D_{t+1}) - \log P_t \]
\[ = p_{t+1} - p_t + \log \left[ 1 + \exp \left( d_{t+1} - p_{t+1} \right) \right] \]
Loglinear approximation

A first-order Taylor expansion around the mean value gives way to

\[ r_{t+1} \approx \alpha + \lambda p_{t+1} + (1 - \lambda) d_{t+1} - p_t, \]

with \( \alpha \equiv (\lambda - 1) \log(1/\lambda - 1) - \log \lambda \) and \( \lambda = \left[ 1 + \exp(d_t - p_t) \right] \)

**US data** average dividend-price ratio is about 4% annually, implying \( \lambda = 0.997 \) in monthly data

\( \rightarrow \) approximation replaces the log of the sum with a weighted average of the logs in which the log price gets most weight for a given proportional change in dividends has a much smaller effect on the return than the same proportional change in price
loglinear approximation gives a linear difference equation for the log stock price that we solve forward imposing the transversality condition that \( \lambda^\kappa p_{t+\kappa} \to 0 \) (i.e., absence of rational bubbles)

\[
\begin{align*}
\hat{p}_t &= \frac{\alpha}{1 - \lambda} + \sum_{\ell=0}^{\infty} \lambda^\ell \left[ (1 - \lambda) d_{t+\ell+1} - r_{t+\ell+1} \right] \\
&= \frac{\alpha}{1 - \lambda} + \sum_{\ell=0}^{\infty} \lambda^\ell \mathbb{E}_t \left[ (1 - \lambda) d_{t+\ell+1} - r_{t+\ell+1} \right]
\end{align*}
\] (ex-post)

\[
\begin{align*}
\hat{p}_t &= \frac{\alpha}{1 - \lambda} + \sum_{\ell=0}^{\infty} \lambda^\ell \left[ (1 - \lambda) d_{t+\ell+1} - r_{t+\ell+1} \right] \\
&= \frac{\alpha}{1 - \lambda} + \sum_{\ell=0}^{\infty} \lambda^\ell \mathbb{E}_t \left[ (1 - \lambda) d_{t+\ell+1} - r_{t+\ell+1} \right]
\end{align*}
\] (ex-ante)

**dividend-price ratio model** (or dynamic Gordon growth model)

\[\rightarrow \text{ effect on price of a high dividend growth rate now depends on how long the dividend growth rate is expected to remain high}\]
Stationary log dividend-price ratio

log dividend-price ratio is high either when expected future dividends grow only slowly or when expected stock returns are high

\[
d_t - p_t = d_t - \frac{\alpha}{1 - \lambda} - \sum_{\ell=0}^{\infty} \lambda^\ell \mathbb{E}_t \left[(1 - \lambda)d_{t+\ell+1} - r_{t+\ell+1}\right]
\]

\[
= -\frac{\alpha}{1 - \lambda} + \sum_{\ell=0}^{\infty} \lambda^\ell \mathbb{E}_t \left[r_{t+\ell+1} - \Delta d_{t+\ell+1}\right]
\]

cointegration analysis is now much easier because we know that the cointegrating vector is \((1, -1)\)
Empirical evidence in US

\[
\sum_{\ell=1}^{\kappa} r_{t+\ell} = \beta(\kappa) (d_t - p_t) + \xi_{\kappa,t+\kappa}
\]

<table>
<thead>
<tr>
<th>forecast horizon ( \kappa ) in months</th>
<th>1</th>
<th>3</th>
<th>12</th>
<th>36</th>
<th>48</th>
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<tbody>
<tr>
<td>1927 to 1951</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta}(\kappa) )</td>
<td>0.024</td>
<td>0.054</td>
<td>0.304</td>
<td>0.925</td>
<td>1.085</td>
</tr>
<tr>
<td>( R^2(\kappa) )</td>
<td>0.007</td>
<td>0.011</td>
<td>0.086</td>
<td>0.330</td>
<td>0.419</td>
</tr>
<tr>
<td>t-stat(( \kappa ))</td>
<td>0.980</td>
<td>0.793</td>
<td>1.915</td>
<td>2.875</td>
<td>3.693</td>
</tr>
<tr>
<td>1952 to 1994</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta}(\kappa) )</td>
<td>0.027</td>
<td>0.080</td>
<td>0.327</td>
<td>0.757</td>
<td>0.843</td>
</tr>
<tr>
<td>( R^2(\kappa) )</td>
<td>0.018</td>
<td>0.049</td>
<td>0.188</td>
<td>0.411</td>
<td>0.417</td>
</tr>
<tr>
<td>t-stat(( \kappa ))</td>
<td>3.118</td>
<td>3.152</td>
<td>3.181</td>
<td>3.280</td>
<td>3.508</td>
</tr>
</tbody>
</table>
Interpretation of the results

predictive ability enhances as the forecast horizon increases because

\[ d_t - p_t = -\frac{\alpha}{1 - \lambda} + \sum_{\ell=0}^{\infty} \lambda^\ell \mathbb{E}_t \left[ r_{t+\ell+1} - \Delta d_{t+\ell+1} \right] \]

shows that

(1) the log dividend-price ratio is a good proxy for expectations of future stock returns only if expectations of future dividend growth rates are not too variable

(2) expectations on the right-hand side are of a discounted value of all returns into the infinite future
Lecture 09: Term structure of interest rates

references
Cuthbertson & Nitzsche, chapters 20 to 22
Campbell, Lo and MacKinlay, chapter 10
Relationship between short and long rates

**motivation** central banks employ short-term interest rates as a lever on the real economy, in an attempt to ultimately influence inflation.

**transmission channel**

(1) changes in short rates may influence real inventory holdings and consumers’ expenditure particularly on durable goods as long as there is no change in inflationary expectations.

(2) interest rates on long maturity government bonds may affect corporate bonds rate, which determines real investment.
if interest rate policy involves a **credible** anti-inflation strategy, then a **rise in short rates** presumably leads to **lower expectations of future inflation** as well as **lower future short-term interest rates**, and hence **lower long rates** \(\rightarrow\) downward sloping yield curve

on the other hand, if nominal interest rates increase in response to a higher level of inflation, then long rates might rise as people expect higher inflation in the future \(\rightarrow\) upward sloping yield curve
Yields and holding period return

**Spot yield**  rate of return that applies to investments on a known risk-free interest rate over a given horizon

**Holding period return**  if one holds a coupon-bearing bond between $t$ and $t + 1$, then return consists of capital gains plus any coupon payment

$$H_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)} - P_t^{(n)}}{P_t^{(n)}} + C_t$$
**Pure expectations hypothesis**  

If agents are risk neutral and care only about expected returns, then the expected holding period return on all bonds, regardless of their maturity, must equal the risk-free one-period interest rate:  

$$E_{t}H_{t+1}^{(n)} = R_{t}$$

**Implication**  
Ex-post excess holding period return is an innovation under rational expectations:  

$$H_{t+1}^{(n)} - R_{t} = \xi_{t}^{(n)}$$

**Risk premium**  
To relax the risk neutrality assumption, we must incorporate a term premium to reward for the risk of holding bonds with longer maturities:  

$$E_{t}H_{t+1}^{(n)} = R_{t} + T_{t}^{(n)}$$
Expectation hypothesis

we must now model the term premium, otherwise \( \mathbb{E}_t H_{t+1}^{(n)} - R_t = T_t^{(n)} \) constitutes a tautology

**restrictions** term premium is constant over time and independent of the maturity of the bond: \( T_t^{(n)} = T \)

**implication** ex-post excess holding period return has nonzero mean, but it still exhibits no serial correlation under rational expectations: \( H_{t+1}^{(n)} - R_t = T + \xi_t^{(n)} \)

**variance inequality** \( \text{var} \left( H_{t+1}^{(n)} \right) \geq \text{var} \left( R_t \right) \) (sanity check)
Liquidity preference hypothesis

**assumption** term premium does not vary over time but it does depend on the term to maturity of the bond: $T_t^{(n)} = T^{(n)}$

liquidity considerations require a term premium that increases with maturity, **though**... the gaps in expected holding period returns across periods to maturity is constant through time

**implication** ex-post excess HPR not only has nonzero mean that depends on time to maturity, but also exhibits no serial correlation under rational expectations

$\rightarrow \quad H_{t+1}^{(n)} - R_t = T^{(n)} + \xi_t^{(n)}$ with $T^{(n)} > T^{(n-1)} > \ldots$
alternative $n$-period investments

(1) $n$-period bond

$$ L^{(n)} = L^{(0)} \left( 1 + R_t^{(n)} \right)^n $$

(2) rolled-over one-period investments for $n$ periods

$$ \mathbb{E}_t L_*^{(n)} = L^{(0)} (1 + R_t) (1 + \mathbb{E}_t R_{t+1}) \cdots (1 + \mathbb{E}_t R_{t+n-1}) $$

risk neutrality then implies that $\mathbb{E}_t L_*^{(n)} = L^{(n)}$, and hence that

$$ \left( 1 + R^{(n)} \right)^n = (1 + R_t) \prod_{j=1}^{n-1} \left( 1 + \mathbb{E}_t R_{t+j} \right) $$
Expected excess return

\[
\mathbb{E}_t \pi_t^{(n)} = R_t^{(n)} - (1 + R_t) \prod_{j=1}^{n-1} (1 + \mathbb{E}_t R_{t+j})
\]

\[
= \begin{cases} 
\text{zero} & \text{under pure expectations hypothesis} \\
\pi_{EH}^{(n)} & \text{under expectations hypothesis} \\
\pi_{LPH}^{(n)} & \text{under liquidity preference hypothesis}
\end{cases}
\]

ignoring Jensen’s inequality as in Campbell (Journal of Finance 1986) yields an approximate linear relationship between long and short rates with continuous compounding: 

\[
r_t^{(n)} \approx \mathbb{E}_t \left( \frac{1}{n} \sum_{j=1}^{n} r_{t+j-1} \right)
\]
**Forward rates**

**motivation** forward rates are riskless investments given that they are measurable at time $t$

**example** two-year investment at $r^{(2)}_t$ must yield the same return as a one-year spot investment at $r_t$ followed by a forward investment between $t$ and $t + 1$, at a forward rate $f_t^{(2|1)}$

\[
f_t^{(2|1)} = 2r^{(2)}_t - r_t = 2\mathbb{E}_t \left( \frac{1}{2} \sum_{j=1}^{2} r_{t+j-1} \right) - r_t = \mathbb{E}_t (r_{t+1})
\]

**main implication of the pure expectations hypothesis**

rational expectations $\longrightarrow$ $r^{(n-m)}_{t+m} - r_t = \alpha_F + \beta_F (f_t^{n|m} - r_t) + \zeta^{(n)}_{t+m}$

with $\alpha_F = 0$ and $\beta_F = 1$
Expected changes in the long rate

starting point expected HPR with continuous compounding

\[ \mathbb{E}_t h_{t+1}^{(n)} = \mathbb{E}_t \left( \log P_{t+1}^{(n-1)} - \log P_t^{(n)} \right) = r_t + T_t^{(n)} \]

\[ P_t^{(n)} = P_0 e^{-r_t^{(n)} n} \Rightarrow \mathbb{E}_t \left( nr_t^{(n)} - (n-1)r_{t+1}^{(n-1)} \right) = r_t + T_t^{(n)} \]

\[ (n-1)\mathbb{E}_t \left( r_t^{(n)} - r_{t+1}^{(n-1)} \right) + r_t^{(n)} = r_t + T_t^{(n)} \]

resulting relation decomposes the expected changes in the long rate into actual spread plus a term premium

\[ (n-1) \left( \mathbb{E}_t r_{t+1}^{(n-1)} - r_t^{(n)} \right) = \left( r_t^{(n)} - r_t \right) - T_t^{(n)} \]
current long rate as weighted average of future short rates

\[ r_t^{(n)} = \mathbb{E}_t \left( \frac{1}{n} \sum_{j=1}^{n} r_{t+j-1} \right) + \frac{1}{n} \sum_{j=1}^{n} T_{t+j-1}^{(n-j-1)} \]

↓ ↓ ↓ ↓ ↓
perfect foresight average risk long rate premium \( \Phi_t^{(n)} \)

future short rates

\[ \sum_{j=1}^{n-1} (1 - j/n) \mathbb{E}_t \Delta r_{t+j} = r_t^{(n)} - r_t + \mathbb{E}_t \Phi_t^{(n)} \]

↓ ↓ ↓ ↓ ↓
perfect foresight actual term spread spread premium
Regression tests

(1) \( r_{t+m}^{(n-m)} - r_t = \alpha_F + \beta_F (f_t^{n|m} - r_t) + \zeta_{t+m}^{(n)} \)  
(pure expectation hypothesis: \( \alpha_F = 0 \) and \( \beta_F = 1 \))

(2) \( (n - 1) \left( r_{t+1}^{(n-1)} - r_t^{(n)} \right) = \alpha_L + \beta_L \left( r_t^{(n)} - r_t \right) + X_t \gamma + \zeta_{t+1} \)  
(expectation hypothesis: \( \beta_L = 1 \))

(3) \( \sum_{j=1}^{n-1} (1 - j/n) \Delta r_{t+j} = \alpha_S + \beta_S \left( r_t^{(n)} - r_t \right) + X_t \gamma + \zeta_{t+n-1} \)  
(expectation hypothesis: \( \beta_S = 1 \))

remark regressions (2) and (3) model the (average) term premium as \( \alpha + X_t \gamma \), coinciding if \( n \) is twofold the sampling frequency
Empirical evidence

(1) long and short rates cointegrate, with cointegrating vector close to (1,-1) → stationary spread

(2) spreads do Granger cause future changes in interest rates for most maturities

(3) there are dynamic nonlinearities in the relationship between interest rates that depends on the level of the spread

(4) expectations hypothesis works rather well for US, especially on the basis of the forward rate regressions, if one excludes the 1979-1982 monetary base control experiment
Lecture 10: Volatility extraction

references

Cuthbertson & Nitzsche, chapter 29
Campbell, Lo and MacKinlay, chapter 12.2
Road map

1. stylized facts vs conditional volatility
2. testing for volatility clustering
3. historical volatility and EWMA
4. GARCH-type models
5. intraday information
Motivation

**stylized facts:** leptokurtosis
   volatility clustering
   leverage effects

**volatility models** not only engender excess kurtosis and volatility clustering, but may also easily accommodate leverage effects

**typology:** stochastic volatility
   mixture of distributions
   EWMA vs GARCH models
Testing for volatility clustering

**visual inspection:** histogram and time series plot
plot of absolute/squared values
recursive historical volatility plots

**statistical tests:** portmanteau tests for absolute/squared values
LM-type tests for absolute/squared values
LM-type tests with leverage effects
1. sample variance of past returns: $\hat{\sigma}_t^2 = \frac{1}{21} \sum_{s=0}^{21} (r_{t-s} - \bar{r})^2$

2. moving average: rolling window controls smoothness inefficient weighting scheme

3. time series properties $\rightarrow$ long-run vs realized variance
Exponentially weighted moving average, EWMA 10.5

**idea:** more weight to recent observations

**solution:** exponential decay

\[ \hat{\sigma}_t^2 = (1 - \lambda) \sum_{s=0}^{t-1} \lambda^{s-1} (r_t - s - \bar{r})^2 \]

1. smoothness parameter \( \lambda \)

2. time series properties \( \rightarrow \) nonstationary
$r_{t+1} = \sigma_{t+1|t} \epsilon_{t+1}$, where $\epsilon_t \sim \text{iid } f(0, 1)$

**ARCH(p):**

$$\sigma_{t+1|t}^2 = \omega + \sum_{s=1}^{p} \alpha_s r_{t-s+1}^2$$

**GARCH(1, 1):**

$$\sigma_{t+1|t}^2 = \omega + \alpha r_t^2 + \beta \sigma_{t|t-1}^2$$

**IGARCH(1, 1):**

$$\sigma_{t+1|t}^2 = \omega + \alpha r_t^2 + (1 - \alpha) \sigma_{t|t-1}^2$$

**EGARCH(1, 1):**

$$\ln \sigma_{t+1|t} = \omega + \alpha |\epsilon_t| - \gamma \epsilon_t + \beta \ln \sigma_{t|t-1}$$

**GJR-GARCH(1, 1):**

$$\sigma_{t+1|t}^2 = \omega + \alpha r_t^2 + \gamma r_t^2 \mathbf{1}(r_t < 0) + \beta \sigma_{t|t-1}^2$$

**estimation:** quasi-maximum likelihood $\rightarrow$ normal distribution
Intraday information

intraday data: high-low data → range-based estimators
regular spacing → realized approach
tick-by-tick → ACD model for price durations

advantages: simplicity
semiparametric nature

disadvantages: posterior dynamic modeling
market microstructure noise