ON WALRASIAN MODELS AND DECENTRALIZED ECONOMICS

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1 INTRODUCTION

One of the main tasks of economic theory is to explain the outcomes of a decentralized economy. The best developed model which addresses this problem is the by now standard, Arrow-Debreu, general equilibrium model (further referred to as the standard Walrasian model). Almost nobody has ever claimed that this model gives a realistic description of a decentralized economy, and the founders of this theory have never had any pretension to realism (see Punzo [1988] for a survey, but also Negishi [1962] for an exception). Hence, in this note we will restrict our attention to the, relatively safe, domain of pure theory. In that field, then, frequently something as the following is claimed. Take an economy consisting of a set $A$ of autonomous agents with given preferences $\succeq \in P$ and endowments $\omega \in \mathbb{R}_+^f: \mathcal{E}: A \rightarrow P \times \mathbb{R}_+^f$. Let these individual agents freely choose their demands, given the prices, then it can be proved that, having made only the appropriate assumptions about the primitives of the economy, there exists an equilibrium in which the choices of all these agents may be realized. Note that this would be a remarkable result, as each individual agent was considering only his own preferences and endowments. In this note we address the question whether such a claim is right.

An answer to this question is of theoretical importance. Within the social sciences there is a continuous debate about the appropriate basic subject-matter of social theory. To simplify matters in economics terms, there is a spectrum of possibilities ranging from the micro to the macro level (see e.g. Giddens & Turner [1987] for a systematic overview). Now, Walrasian economists claim that they can explain both the behavior of individual agents and the overall outcome of their actions in a decentralized economy, by adopting the methodological point of view of the first of the extremes of the spectrum, that is by starting their analysis at the level of autonomous subjects. An autonomous subject is an agent whose set of possible actions and outcomes is not predetermined by any form of a given structure, a set of rules, a certain context, or anything that transcends the level of
the agents. Moreover, a theory which considers the overall outcome of these individual actions is a theory based upon autonomous subjects if and only if this overall outcome is, in one way or another, thought to depend only upon the behavior of these agents. If the Walrasian claim would be right, then that would be an important theoretical achievement.

But an answer to the question is also of practical importance. For example, a popular idea among both economists and policy makers is that the purely theoretical, mathematical economics, although dealing with unrealistic and abstract models, has at least proved that a decentralized economy is optimal, as far as there are no real world complications in the form of external effects, public goods, increasing returns, etc. Such an idea may lead both researchers (e.g. Hahn [1982]) and policy makers to focus attention exclusively upon the problem of such complications. However, if the Walrasian claim would be unjustified, then statements about the acceptability or optimality of decentralized trade cannot be defended at all by referring to theoretical results concerning Walrasian models.

We will argue, however, that the claim is not right. In section 2 we describe the Walrasian point of view on how to model a decentralized economy. We expose the structure not only of the standard Walrasian model, but also of fixed price models, imperfectly competitive models and temporary equilibrium models. Although that is not an exhaustive survey of models which exhibit the same structure, it will suffice to indicate the implications and limitations of the Walrasian approach to the modeling of a decentralized economy, and to evaluate the claim in section 3. There we argue that the structure of Walrasian models is such that the Walrasian claim is inconsistent, and that one cannot consider these models as ideal representations of decentralized economies. In section 4, by discussing some other Walrasian views, we anticipate some possible criticisms of Walrasians who will be in disagreement with the analysis of the previous sections. In section 5 we conclude by indicating some possibilities of future developments of economic theory. We argue that a different methodological point of view, paying attention explicitly to the interaction between individual agents in a decentralized economy, is important.

2 THE WALRASIAN PERSPECTIVE

A description of the Walrasian approach to the modeling of decentralized economies may seem superfluous, but it turns out sometimes that economists do not know exactly what is implied by this perspective. Usually, the adjective Walrasian is related to market-clearing equilibrium in all markets, which is accomplished by fully flexible prices. Sometimes even something like an auctioneer-cum-tatonnement
construction is mentioned, but, for example, the fact that the auctioneer and the \textit{tâtonnement} process are two logically separate concepts is not always understood (e.g. Laroque [1987] or Benassy [1987]). Non-Walrasian is then simply thought as less than fully flexible prices, resulting in non-market-clearing, i.e. non-Walrasian, equilibria. Such models are often called non-Walrasian models. However, we will show that these models have the same general structure as the standard Walrasian model, and that they reveal a common approach to or view on how to model a decentralized economy. This methodological point of view we will call the Walrasian perspective.

In the standard Walrasian story one starts with the existence of an auctioneer. He publicly announces prices for every good. Taking into account endowments, preferences, and technologies, each optimizing agent, believing the economy is in equilibrium, declares him the amount of each good he wants to demand at the announced prices. If the auctioneer, after aggregation over all individual agents, finds out that some market excess demands are not equal to zero, he applies a simple price adjustment rule, changing prices proportionally to the aggregate excess demand, so that if excess demand is positive he raises prices. Given these new prices agents express their revised plans to the auctioneer, who considers them again. This process of groping (\textit{tâtonnement}) continues until the auctioneer has found the vector of prices at which excess demands equal zero in each market. Only then transactions take place and every agent will indeed be able to transact exactly as much as he planned at the given prices. When all the transactions have been executed, the time for consumption and production starts. The future can be divided in a finite number of elementary periods and 'states of the world', and markets for all commodities in all future periods and all 'states of the world' exist at one point in time, i.e. at the beginning of the economic history. Contracts will be concluded at that moment. In the future they only have to be executed.

In order to show the meaning of the Walrasian perspective further and to illustrate its logical distinction from a Walrasian equilibrium as such, we now discuss some classes of models which may yield non-Walrasian equilibria, and which are often, slightly misleadingly, called non-Walrasian models.

A development which has attracted much attention since the beginning of the 1970s is the literature about models in which prices are fixed (e.g. Barro & Grossman [1971] or Benassy [1982]). In these models an auctioneer announces a vector of fixed prices which is not necessarily equal to the Walrasian equilibrium one. Given the price vector, each individual agent, taking into account his endowments and maximizing utility, expresses a vector of demands (for all markets simultaneously), which may be called 'notional' demands. Depending upon the
expressed demands of all the other agents in the model and given a set of functions which relates these notional demands to attainable transactions for every agent (i.e. a set of rationing schemes), each individual agent might hear that he would be constrained in his transactions in some markets. Taking into account these quantity signals, every agent may express revised demands, effective demands, for all markets, and consequently hear changed constraints. The auctioneer continues this quantity tatonnement process until the newly expressed effective demands are equal to the former ones. Only then, when a K(eynesian) equilibrium has been reached, transactions may take place. As a result, the perceived quantity constraints which the agents have taken into account when determining their final effective demands are the same that will actually be generated by the exchange process. Generally, it is assumed that the rationing schemes have at least the following properties: Voluntary exchange and market efficiency (i.e. if there is aggregate excess demand for a good, then no agent can have an unsatisfied supply of that good, and vice versa), which, taken together, provide the 'short-side rule'. This rule, which says that the 'short side' of the market will always be able to realize its demand, implicitly presumes the performance of an auctioneer.

Note that in the resulting K-equilibrium there may be aggregate excess demands not equal to zero. In this sense there is no perfect coordination as in a Walrasian equilibrium. However, given the institutionally restricted space of prices, in a K-equilibrium every agent gets exactly what he expected when expressing his demands, and in this sense the plans of all agents are compatible.

A third class within the Walrasian perspective concerns imperfectly competitive models, in which "at least one agent in the economy has the right knowledge that the signals which he receives from his environment depend in whole or partially upon his own choices in a foreseeable way, and tries to exploit this dependence to his own profit" (Gary-Bobo [1987], p. 2).

In a Cournot-Walras model (Gabszewicz & Vial [1972]) firms use quantities as a strategic variable, exploiting their objective knowledge of the demand function. Firms are assumed to know that, and how, a change in their own proposed demands influences the price vector. For the rest, the tatonnement proceeds as usual. The theory of general equilibrium with price making was first developed by Negishi [1961]. The most general treatments are now found in Benassy [1982] and Benassy [1987]. The former combines the theory of Negishi [1961] with the more recent theory about rationing. Commodities are distinguished by the agent who sets its price, and a subset of prices is assumed to be fixed. Each price maker may choose the prices of a subset of commodities in order to manipulate his quantity constraints. Given the perceived demand curve a price maker will set his price by solving the
usual optimization problem, thus making equal marginal cost and marginal revenue. However, the quantity signals used to estimate the demand curve and the generation of these quantity signals by setting prices are, in fact, two simultaneous, interacting processes. To solve this problem Benassy [1982] makes play with the cryptic description of an "implicit instantaneous interaction" (p. 95) of two processes. In other words, Benassy relies upon an auctioneer-cum-tatonnement process in both prices and quantities. A K-equilibrium has been reached when every price maker is satisfied with all the price-quantity combinations obtained, and thus doesn't want to change the prices of the subset of commodities of which he is a price maker, or the quantities of the other subset of commodities of which he is a price taker. Only then transactions take place. This procedure assures that perceived constraints are equal to actual constraints in equilibrium, thus satisfying the minimal coherence condition of the subjective approach. In the 'objective' version of this approach, perceived and actual demand curves are equal everywhere. Such an analysis is performed by Benassy [1987] who makes use of a dual tatonnement as well, although he claims that he has given an analysis of "price making by decentralized agents in the absence of an auctioneer" (p. 23).

Dynamic models within the Walrasian perspective are the result of the development of temporary equilibrium models; an idea usually attributed to Hicks [1939] and brought into fashion in the 1970s by Grandmont. These models can be characterized by two modifications of the models discussed above. First, the set of future markets is not complete, i.e. not every conceivable forward commitment is possible. To determine an equilibrium resort is taken to one of the Walrasian auctioneer-cum-tatonnement mechanisms as outlined above. That is, prices may be flexible, fixed or set by price makers, and thus an equilibrium may be Walrasian or, for example, K(eynesian). This concept of equilibrium only applies to a single period and is called a 'temporary equilibrium'. Because a non-empty part of the space of goods is unmarketable, at the beginning of time not all transactions can be executed, and not all the plans of all the agents are pre-reconciled. This has two interesting consequences. First, agents may take the a priori given transaction constraints with respect to some future goods into account when deciding on their plans concerning other goods. Second, the individual agent's decisions depend not only upon current variables, but also upon his expectations concerning all not-yet-determined future variables. Thus, second modification, the temporary equilibrium model concerning

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1 According to Benassy [1982] this ambiguous feature is inherent to the use of this framework and can be found, explicitly or implicitly, in all the literature on equilibria with monopolistic competition.
only one period logically requires an extension to future periods. This is done by considering a sequence of temporary equilibrium models. In each elementary period markets have to reopen because not all forward commitments have been possible in preceding periods. Only at discrete intervals, at the junction of subsequent periods all agents simultaneously make new decisions, taking account of the decisions of the past and their consequences. Within each period only consumption and production as a result of concluded commitments take place. In fact, as Grandmont [1977] puts it: "Once an equilibrium is reached, trading takes place, and the economy moves to the next period" (p. 557).

Without discussing the relative merits or weaknesses of the reviewed models, we can conclude that all sketched models have, explicitly or implicitly, the following properties. First, the existence of an omniscient and omnipotent auctioneer. The auctioneer collects and disseminates information, he adjusts prices and/or quantities, and he physically executes the actual exchange at the appropriate moment, thus taking care of a frictionless, orderly market. Moreover, the auctioneer must check that the rules of the game are respected; thus e.g. each individual’s budget constraint must be obeyed. Second, the game this auctioneer plays in order to execute his functions is mainly a tatonnement process. Only when this groping doesn’t provide the auctioneer with new information, exchange may take place, but never before that moment. If transactions would take place before, endowments would be changing and so would the demands. Third, time plays no real role. There is assumed to be a conceptual separation between the time in which adjustment and finally exchange take place (the auctioneer’s time, or meta-time) and the time in which commodities are dated and consumption and production occur (the agents’ time, or real time). This implies that adjustments may be thought as taking place immediately. Fourth, as a result, all agents are justified ex post in expressing their choices under the assumption that they will never be constrained more in their actions than expected ex ante; in other words, under the assumption that the economy is in equilibrium. Fifth, as soon as the processes taking place in meta-time are finished, and an equilibrium has been found, all realizable exchanges are executed immediately and the end of the economic history has been reached. All that rests is the enjoyment of the attained commodities or the obligation to fulfill

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2 In a fixed price temporary equilibrium model the auctioneer might change prices at the junction of two periods without a tatonnement process.

3 A discussion of the literature taking up this possibility (e.g. Hahn & Negishi [1962]) is beyond the scope of this paper.
the accepted commitments. Equilibrium (Walrasian or non-Walrasian) in this perspective is always a terminal state.\(^4\)

### 3 APPRAISAL OF THE WALRASIAN PERSPECTIVE

Now we are in the position to answer the question presented in the introduction. The answer must be negative, i.e. the claim is not right. The Walrasian perspective leads to models which are paradoxical. They pretend to describe a decentralized economy, by basing their analysis upon the actions of autonomous agents. That is, besides making assumptions about the primitives of the economy, they pretend to discard each kind of external determination of the behavior of the individual agents. However, to explain both the individual actions and the overall outcome, in which all individual choices may be realized as planned, it turns out that resort has to be taken to concepts and structures which transcend the level of the individual agents. Hence, the Walrasian claim is inconsistent.

The following points sustain this conclusion.\(^5\) First, the existence of the auctioneer, the division of time into meta- and real time, and the rules of the game in these models are in no way the product of the behavior of autonomous agents. Note that every agent is assumed to exhibit optimizing behavior, but what is the auctioneer optimizing? Why should he, for example, change any price at all? (see e.g. Weintraub [1979]). Secondly, although not the actions of the individuals are predetermined (they may choose to express the demands or prices they individually prefer), their set of possible actions is predetermined by the rules and structure of the model. Depending upon the variant, the only thing individual agents may do is to express some demands or prices to the auctioneer, but in no case are they allowed to trade, consume or produce without his explicit permission, and never will they get permission for any form of interaction among each other. Thus, the individual agents are not autonomous subjects. Thirdly, whatever the phase of the tâtonnement process, each individual in calculating his choices takes the structure of the model into account, trusting the overall outcome to be such that he will anyway be able to trade as much as he plans. Clearly, the overall outcome still depends upon the preferences of the individual agents, but these agents are

\(^4\) Temporary equilibrium models partially form an exception in this respect in the sense that each terminal state is temporary.

\(^5\) The first of these points is the best known and therefore, although important, relatively trivial to make. But note that we do not argue in terms of 'lack of realism'. The second and third point are, however, at least as important as the first, and may deserve some special attention.
assumed to *anticipate* the equilibrium character of the overall outcome, which itself should, instead, be explained by their actions. How could an individual agent in this model get the idea that the economy will turn out to be in equilibrium (unless he is God, the man with the invisible hand or the auctioneer)? Thus, Adam Smith's transcendental hand has been made visible. But it is still there.\(^6\)

### 4 OTHER WALRASIAN VIEWS

It should be noted that some authors have argued that the link between the discussed perspective and the name of Walras doesn't do complete justice to the latter's insights (see e.g. Morishima [1977] or, of course, Walras [1874]). Moreover, not every Walrasian economist would like the *auctioneer-cum-tâtonnement* stories as told above.

Monetarist and new classical economists, for example, would prefer a more 'realistic' story about individual agents wandering about, perceiving and pursuing every advantageous opportunity. However, if it is assumed that individual agents understand the full state of affairs in all markets simultaneously, an omniscience and calculating ability is imputed to agents which seems characteristic only of a Walrasian *auctioneer-cum-tâtonnement* construction (see e.g. Fisher [1983]). And indeed, both in monetarist and new classical theories the natural levels of employment and production are supposed to be determined by a Walrasian model, taking into account some real world frictions. Consider, for example, the following quotation of Friedman [1969]: "The 'natural rate of unemployment' ... is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual characteristics of the labor and commodity markets" (p. 102). Also Sargent considers the economy as organized by "something that operates as a Walrasian auctioneer" (Klamer [1984], p. 69). Clearly, such positions do not help to sustain the claim analyzed in this note either.

A group of Walrasian theoretical economists, instead, would say that nowadays it is clear that the Walrasian perspective gives some *unresolvable* technical problems (see Kirman [1989] for a survey). In one sentence, it is theoretically impossible to get needed characteristics of aggregate demand functions (needed in

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\(^6\) A word of warning is in order here. The decisive point to reject the Walrasian claim is not the story of adjustment to equilibrium. If we abstract from the process of adjustment, i.e. if the auctioneer would just announce the equilibrium values of the relevant variables, the whole analysis remains of importance. It is the structure implied by the Walrasian perspective as such which is the point.
order to prove stability of the *tâtonnement* process) by imposing more and more restrictions upon individual characteristics. Because of this aggregation problem, the Walrasian claim to consider should be stated more precisely. Usually referring to Debreu [1959], these economists only claim the following. Take an economy \( \mathcal{E} : A \rightarrow P \times \mathbb{R}_+^l \), let the individual agents, for each price vector \( p \), determine their demands by choosing the best point in their budget set, then, having made only the appropriate assumptions about the primitives of the economy, there exists a price vector \( p^* \) such that in each market the sum of the chosen quantities equals the total available resources. Thus, no auctioneer or *tâtonnement* enter the scene, and nothing is said about the structure of the economy. Debreu [1959] gives a mathematical proof of such a claim.

However, it seems that the meaning of this proof of existence of equilibrium, needs a careful consideration. It is a widespread belief that in Debreu’s model it is assumed that all agents are able to communicate and trade freely with each other, but attentive readers will notice that words as communication and trade are not even mentioned once in Debreu [1959]. There is *supposed* to be a (central) price vector \( p \), and all individual agents are *supposed* to know this, but it is not explained that and how this will be so. Individual agents are *supposed* to determine their demands by choosing the best point in their budget set. That is, the notion 'action' is *a priori* taken as synonymous to the notions 'best point in a budget set' and 'demand', but is is not explained that these notions will indeed be equivalent for each individual agent. Why should agents choose the best point in their budget set as demands, and why would they not take any other action? Moreover, in Debreu [1959] trade as such is not considered at all. That is, individuals choose the best point in their budget set as demands, but in no way, whatsoever, the question of how these demands might be realized is considered. This is important, because it is with respect to these realizations that a number of problems would enter the scene (see e.g. Ostroy [1973]).

Thus, Debreu [1959] is not a theory which explains the individual actions or the overall outcome in a decentralized economy, and it still asks for a theory of communication and a theory of trade in such an economy. In this sense the Debreuian version of the Walrasian claim is much more modest, and therefore it is not right to say, as is sometimes done, that Debreu finally proved mathematically what Smith argued almost two centuries before. Nevertheless, Debreu gives an important result which may serve as a bench mark.
5 CONCLUDING REMARKS

Having learned much from the implications, limitations and problems of the Walrasian perspective, and assuming that it is important to have theories about decentralized economies which are as sophisticated as the Walrasian models, there seem to be, roughly, three possible conclusions so far as future developments of theories of decentralized economies are concerned.7

First, to ignore these problems and limitations, continuing with models and equilibrium concepts which implicitly assume a Walrasian structure, and with representative agents, which assumes away the aggregation problem. Secondly, to forget about the individual agents and the ways one could derive aggregate conclusions from their behavior, focussing, instead, upon macroeconomics as a purely empirical discipline: just observing regularities in aggregates (see e.g. Fitoussi [1983]). However, leaving out of consideration the details, the impossibility of empirical knowledge independent of any theoretical structuring is well-known. As Hahn [1983] puts it: "If we are interested in the behavior of aggregates then we must use economic theory to help us, and the only theory we have is one of rational and self-seeking agents" (p. 223). Therefore, third possibility, it seems necessary to change perspective, and to adopt a different view on how to model a decentralized economy. Thus, starting at the level of individual agents, one should not try to model them as autonomous agents. If one wants to derive conclusions concerning the overall outcomes of the economy, without wanting to resort to the Walrasian 'star',8 it seems that one should allow for some forms of interaction between the individual agents. All kinds of information and coordination problems have to be handled by the individuals themselves, and questions arise concerning the influence of decisions of individual agents upon other agents. Each individual’s activities will in a certain way be 'involved' in the activities and decisions of some other agents. Thus, each agent will have a different relevant 'environment' for different kinds of activities. Individuals are not autonomous agents in the sense that their possible actions and outcomes do depend upon their given relevant environment. On the other hand, there is no fixed structure of these relevant environments as each individual action might influence the environments of some individuals, and it is

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7 Of course there do exist other models of decentralized trade than the Walrasian models discussed here, but this literature (e.g. Fisher [1983] or Goldman & Starr [1982]) deserves a separate discussion.

8 This term refers both to the leading part of the auctioneer and to the geometrical structure of communication and trade in Walrasian models.
impossible to attribute the existing structures to one subject, or to explain it by one big reason, because they are the result of innumerable big and little decisions by individuals which gear into each other and may produce results which were not in the mind of any of these individuals. 9

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REFERENCES


9 This rather sketchy indication of a methodological point of view which might be more fruitful in order to describe a decentralized economy is more or less based upon Veyne [1978]. It seems to come close to what Prychitko [1989] argues to be the methodological point of view of the "sophisticated individualism" of Hayek and the Austrian School.


Hicks, J.R. (1939), *Value and Capital*, Oxford University Press.


