WAGE DYNAMICS, COHORT EFFECTS, AND LIMITED COMMITMENT MODELS

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Abstract
In this paper we analyse a model in which firms cannot pay discriminate based on year of entry to the firm, and argue that the wage dynamics are consistent with the empirical results of Beaudry and DiNardo (1991). Their results have been interpreted as supporting a model in which workers are ex post mobile. Since in our model worker mobility/commitment does not affect the optimal contract, it is argued that existing empirical research does not discriminate between different models of worker commitment. (JEL: E32, J41)

1. Introduction

The paper reanalyses a problem first addressed by Beaudry and DiNardo (1991) (hereafater BDN). They develop three versions of a model of labour contracting where a risk-neutral firm potentially offers insurance to risk-averse employees. In the spot market model, wages are determined solely by the value of a worker's marginal product, in the full-commitment contracting model, wages are constant, and in a version with no worker commitment (perfect mobility), where the worker is free to quit at any point, they show that the wage follows a ratchet like process, rising whenever the labour market is tighter than hitherto (since the worker joined the firm), but staying constant otherwise; hence the current wage is determined by the tightest labour market during a worker's tenure. They test these three models against each other on U.S. data. Perhaps surprisingly, the perfect mobility model appears to perform much better than the other two. Subsequent research (McDonald and Worswick 1999; Grant 2003; Shin and Shin 2003) has largely confirmed these results over different periods. The literature has taken this work as indicating two things: First, that implicit contracting combined with ex post mobility may be an important factor driving real wages over the business cycle; and second that

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"cohort" effects are important (Gibbons and Waldman 2003), in that the preferred model implies that workers are treated differently depending on when they are hired, at least until the labour market tightens sufficiently. In this paper, we want to reexamine these conclusions.

We consider a model with a similar basic environment to that considered by BDN (risk-neutral firms, risk-averse workers, fluctuating outside opportunities for workers), but that imposes an equal treatment condition. In other words, a firm cannot pay different workers different wages: all workers must be paid the same irrespective of when they were hired.[1] This implies that a firm cannot treat each entry cohort separately in devising its wage contract, but must fit new hires into an existing wage structure—there are "no cohort effects" in usual parlance. It should also be pointed out that such a model has radically different macro-economic properties from those considered in BDN: The optimal wage structure does not imply that wages are always at a level that makes new hires indifferent about joining a firm—wages have an efficiency wage flavour in that the firm has to trade-off cheaper new hires against savings from offering incumbents stable wages.

Given equal treatment, worker mobility/commitment does not affect the optimal contract under the assumption (which we impose) that the firm needs to hire in each period. The intuition here is straightforward: If the firm desires to hire then it must offer outsiders at least their reservation utilities, and if it cannot discriminate in favour or against incumbents, then it follows that they too must be offered continuation utilities that do not lie below the outside determined reservation utilities. This is precisely the condition on continuation utilities that is usually imposed when there is no worker commitment, so the optimal contract in our model will prevent workers from leaving even if there is costless mobility, and a fortiori will prevent them from leaving if mobility is costly or there is some commitment on their part.

The fact that equal treatment leads to the ex post mobility conditions being satisfied suggests that our model might lead to similar dynamics to BDN's mobility model (and hence might be similarly consistent with their tests). We argue that this is so, and that the tests conducted by BDN and others do not discriminate between these two models. Thus we argue that the existing empirical research cannot be interpreted as evidence in favour either of worker mobility or the existence of cohort effects.

2. A Model with Equal Treatment

We consider the problem faced by a single firm.[2] The model is as follows. There is an infinite horizon $t = 1, 2, 3 \ldots$. All workers are assumed to be identical, and

1. All workers have the same productivity, and thus once in the firm are ex post identical.

2. Since the testable implications derived in BDN depend only on this level of analysis, it is sufficient for our purposes. See Martins, Snell, and Thomas (2004) for a general equilibrium analysis.

we abstract from any tenure or experience effects. Workers are risk averse with per period utility function $u(w)$, $u' > 0$, $u'' < 0$, $\lim_{w\to 0} u(w) = -\infty$, where w is income received within the period; it is assumed that they cannot make credit market transactions. The firm is risk neutral. Workers and firms discount the future with common discount factor $\beta\epsilon(0, 1)$. Workers have a probability $\delta\epsilon(0, 1)$ of survival each period. We assume that the firm has a desired employment level each period that is fixed at N, where we treat N as being large, and hence the number of workers needing to be replaced is taken to equal $(1 - \delta)N$.

Both firm and workers are assumed to be committed to contracts (although, as noted above, the degree of worker commitment does not in fact affect the optimal contract). The labour market offers a worker currently looking for work (at the start of t) a utility (discounted to t) of $\chi_t = \chi(s_t)$ where s_t is the state of nature at t. We assume that the firm can hire any number of workers by offering at least χ_t (and cannot hire otherwise). The state of nature s_t is assumed to follow a Markov process, with initial value s_1, and state space S (not necessarily finite), but assume that from any state s only a finite number of states $r \in S$ are reachable next period with transition probabilities: $\pi_{sr} > 0$. Let $h_t \equiv (s_1, s_2, \ldots, s_t)$ be the history at t.

The firm offers workers in Period 1 a wage contract $(w_1(h_1), w_2(h_2), w_3(h_3), \ldots)$.[3] We assume equal treatment: A worker joining subsequently, at τ after history h_τ, is offered a continuation of this same contract: $(w_\tau(h_\tau), w_{\tau+1}(h_\tau, s_{\tau+1}), w_{\tau+2}(h_\tau, s_{\tau+1}, s_{\tau+2}), \ldots)$, so that at any date t, all workers in the firm receive the same wage. Let $V_t(h_t)$ denote the continuation utility from t onwards from the contract:

$$V_t(h_t) = E\left[\sum_{t'=t}^{\infty} (\beta\delta)^{t'-t} u(w_{t'}(h_{t'}))|h_t\right]. \tag{1}$$

Under the fixed employment assumption, the firm's problem is reduced to minimizing its costs. Consequently the problem faced by the firm is:

$$\min_{(w_t(h_t))_{t=1}^{\infty}} E\left[\sum_{t=1}^{\infty} (\beta)^{t-1} N w_t(h_t)\right]$$

subject to

$$V_t(h_t) \geq \chi(s_t) \tag{2}$$

for all h_t, $t \geq 1$. Equation (2) embodies the assumption that the firm must hire $(1-\delta)N > 0$ workers every period; it is a participation constraint that says that at any point in the future the contract must offer at least the market utility otherwise the required new hires cannot be made.

3. Thus the wage is assumed to be contingent only on the history of states. If the proportion of leavers each period was random—which we have essentially ruled out by assuming N large—then the contract would be improved if it could condition on this proportion, too.

The optimal contract can be characterised with the help of a simple variational argument. Suppose we are at h_t; by optimality there can be no change to this contract that satisfies all participation constraints and that increases profits. Thus consider, starting from the optimal contract, reshuffling wages between t and $t+1$ in state s, to backload them. Increase the wage at $t+1$ after state s by a small amount Δ, and cut the wage at t by x so as to leave the worker indifferent; do not change the contract otherwise:

$$\pi_{s_t s}\delta\beta u'(w_{t+1}(h_t, s))\Delta - u'(w_t(h_t))x \simeq 0.$$

This backloading satisfies all participation constraints since worker utility rises at $t+1$, and so from this point on constraints are satisfied, but also after h_t and earlier since utility is held constant over the two periods. The change in profits (viewed from h_t) is

$$N(-\pi_{s_t s}\beta\Delta + x) \simeq N\left(-\pi_{s_t s}\beta\Delta + \frac{\pi_{s_t s}\delta\beta u'(w_{t+1}(h_t, s))\Delta}{u'(w_t(h_t))}\right),$$

which is positive for Δ small enough unless

$$\frac{u'(w_{t+1}(h_t, s))}{u'(w_t(h_t))} \leq \frac{1}{\delta}. \tag{3}$$

Since the change in profits cannot be positive by optimality of the original contract, equation (3) must hold: marginal utility growth cannot exceed a certain amount. Conversely, the reverse argument (frontloading), which would be profitable if the strict version of equation (3) holds, cannot be undertaken (only) if next period's participation constraint binds since utility falls at $t+1$, so the constraint would be violated. We summarise in the key lemma:

LEMMA 1. *In an optimal contract with perfect mobility, equation (3) must hold; it can only hold strictly ($<$) if the participation constraint binds at* (h_t, s).

A way, then, to think about the evolution of an optimal contract is that there is a "desired marginal utility (gross) growth rate":

$$\frac{u'(w_{t+1}(h_t, s))}{u'(w_t(h_t))} = \frac{1}{\delta}, \tag{4}$$

which will be maintained, unless a binding participation constraint at $t+1$ forces it to be lower.

For purposes of clarity we assume for the remainder of the paper that $u(\cdot) = \log(\cdot)$. Equation (4) now implies that

$$\frac{w_{t+1}}{w_t} = \delta \tag{5}$$

unless the participation constraint binds at $t + 1$, in which case $w_{t+1}/w_t > \delta$ is possible. It is instructive to compare this with the BDN model (in this context, and with symmetric discounting). The corresponding (gross) "growth rate" in their model is 1: wages stay constant. The only difference arises because the exogenous separation (death) rate for the worker now matters. The reason is the following: if each worker is treated independently as in BDN, then the exogenous separation probability affects firm and worker equally—the firm only has to pay the agreed upon wage next period with probability δ (times $\pi_{s_t s_{t+1}}$) and the worker only receives the wage with the same probability. In the equal treatment model with fixed employment levels, if the worker needs to be replaced, the replacement will receive the same wage as that worker would have; hence the future wage is taken into account with probability one (times $\pi_{s_t s_{t+1}}$) by the firm, whereas the worker still discounts with the separation probability.

We can look at this more from the point of view of hiring costs. In our model, even if new workers (at $t + 1$) can be brought in at a low wage (i.e., when the participation constraint is not binding), because of equal treatment the firm may choose not to cut wages as far as it could (until the constraint binds). The reason is that this would create too much wage variability for incumbents, who would need to be compensated by higher wages at t. The firm has to trade-off the extra premium to compensate by incumbents against the savings made on bringing in new hires at a lower wage. If δ is small, so that many workers are expected to leave, new hires are more important and so this moves the trade-off more in favour of cutting wages. In BDN, since each worker is treated independently by the firm, only the wage variability issue is present, and so wages are held constant when possible.

Next, we need to characterise more precisely what happens to the wage when the participation constraint binds. Note that in an optimal contract $(w_t(h_t))_{t=1}^{\infty}$ the participation constraint binds at the initial date $(t = 1) : V_1(s_1) = \chi(s_1)$. If it did not, the firm would increase profits by cutting $w_1(s_1)$ holding the remainder of the contract fixed, and would still satisfy all participation constraints. We define $\underline{w}_s = w_1(s)$, i.e., the Period 1 wage specified by an optimal contract starting in state s, which delivers exactly χ_s. It must be unique by a simple convexity argument (see below). A key observation is the following: it must be optimal at any date t in state s to set $w_t = \underline{w}_s$ whenever $V_t(h_t) = \chi(s)$. This follows from the fact that the future distribution over states depends only on s, and that the continuation contract must itself be optimal (otherwise replacing the continuation contract by a lower cost one which delivered the same continuation utility would reduce the initial costs but satisfy all participation constraints). Thus, $\underline{w}_s$ is the wage in state s at any t if the participation constraint is binding.

PROPOSITION 1. *An optimal contract evolves according to the following updating rule. With each state $s \in S$ is associated a minimum wage $\underline{w}_s$ such that in* *an optimal contract if at date $t+1$ state s occurs then w_{t+1} is updated from w_t by*

$$w_{t+1} = \max\{\delta w_t, \underline{w}_s\}. \tag{6}$$

Proof. We start by showing that $\underline{w}_s$ is unique. Suppose otherwise: Then there are two distinct contracts that deliver $\chi(s)$ to a worker, both of which satisfy participation constraints and yield the same costs. Take a strict convex combination of these two contracts (i.e., a convex combination of wages at each h_t). From equation (1) and the concavity of $u(\cdot)$ it is clear this increases a worker's utility, and satisfies the participation constraint at each point. Costs are linear in wages, and hence are unchanged. Thus a small reduction in the initial wage (in state s) will still satisfy participation, and will lead to lower costs, a contradiction. So $\underline{w}_s$ is unique. Next, we establish equation (6). Suppose that $\delta w_t > \underline{w}_s$. If the participation constraint at $t+1$ in state s binds, $w_{t+1} = \underline{w}_s$, which contradicts backloading from Lemma 1 (wage growth would be less than δ). Thus the constraint does not hold, so by equation (5), $w_{t+1} = \delta w_t$. Conversely, if $\delta w_t \leq \underline{w}_s$, then if the constraint does not hold, by equation (5) $w_{t+1} \leq \underline{w}_s$. However, it is straightforward to show that $V_{t+1} > \chi_s$ would imply that $w_{t+1} > \underline{w}_s$, a contradiction. So the constraint binds and $w_{t+1} > \underline{w}_s$. □

Thus wages evolve in a simple fashion: They change at the "optimal" rate δ unless this takes the wage below the minimum wage for the current state, $\underline{w}_s$, in which case the wage is fixed at this minimum level. The only thing remaining to be determined is the initial wage, which as remarked earlier, is set at $\underline{w}_{s_1}$.

REMARK 3. *This method of argument can also be used to solve BDN's perfect mobility model, and Proposition 1 applies with the change that δ needs to be replaced by 1. This then implies their ratchet characterization.*

3. Empirical Discrimination

The BDN mobility model has performed well empirically. In this section we argue that for the tests that have been undertaken in the literature, our model has similar predictions. In BDN's model, it is the tightest labour market in the current tenure that determines a worker's current wages. In their (and subsequent) empirical work, the unemployment rate is used as a measure of labour market tightness. BDN found that in regressions for individual real wages that include the contemporaneous unemployment rate, the rate at start of current tenure, and the minimum rate over the tenure to date, it is the minimum rate that tends to have the largest coefficient, and often is the only cyclical variable of the three to be significant, in accordance with their theory. We find below that a similar property is true of regressions based on a simulated version of our model. Consequently these tests do not discriminate between the two models. Nevertheless the models do not make identical predictions, and we discuss some possible ways of discriminating between them.

3.1. An Example

The purpose of the following example is to create simulated data from our model on which a regression of the type used by BDN and others can be run. We are particularly interested in the extent to which the minimum unemployment rate during a worker's current tenure can help predict wages.

We assume that the outside opportunity χ_t can be modeled as the discounted value of an outside wage a_t, the logarithm of which is assumed to follow a random walk:

$$\log a_t = \log a_{t-1} + \varepsilon_t,$$

where ε_t is i.i.d. and mean zero, and a_1 is given. We maintain the assumption that $u(w) = \log(w)$ and identify s_t with a_t. Thus $\chi(a_t) = (1 - \delta\beta)^{-1} \log a_t$.

The problem is simplified because it can be shown straightforwardly that $\underline{w}(a) = ka$ for some constant $k \leq 1$, where we write $\underline{w}(a)$ for $\underline{w}_s$ (the wage payment in the first period of the contract that delivers $\chi(a)$). In logs this is $\log \underline{w}(a) = \log a - z$ for some constant $z \geq 0$. Since at Date 1 the initial workers are constrained, $\log w_1 = \log a_1 - z$ and updating (from equation (6)) becomes $\log w_{t+1} = \max\{\log w_t - g, \log a_{t+1} - z\}$ where $g \equiv -\log \delta$. So the dynamics of $\log w_t$ do not depend on the value of z (apart from an additive constant), which will therefore not affect the coefficient estimates of interest in the regression.

We ran simulations of this model with $\varepsilon_t = -\varepsilon$ or ε with probability 0.5.[4] The contract wage will diverge from the outside option process a_t if and only if $g < \varepsilon$, since for $g \geq \varepsilon$ the updating rule implies $\log w_t = \log a_t - z$, and $z > 0$ would imply the participation constraint is violated. In the absence of a model articulating a link between unemployment and labour market tightness, we simply take a_t as our measure of labour market tightness. We regressed (log) wages w_t on a constant plus the following values of (log) a: current, eight-year lagged, and maximum over the previous eight years.[5] The results are presented in the left panel of Figure 1, where coefficient estimates[6] are graphed against g/ε (this ratio determines the dynamics of the model up to a scaling factor). Lower g/ε corresponds to a smaller desired change in wages relative to the step size of

4. A sufficient condition for existence of an optimal solution is that $\beta < e^{-\varepsilon}$.

5. For $t = 100$ to 200, so that the effect of the initial (binding) constraint would be unimportant. Eight years is approximately average tenure in the data used by BDN.

6. To create a panel, as in BDN, each data run is viewed as data on a single individual, and 1000 independent runs were performed (for each value of g/ε). Standard errors are of the order of 0.001.

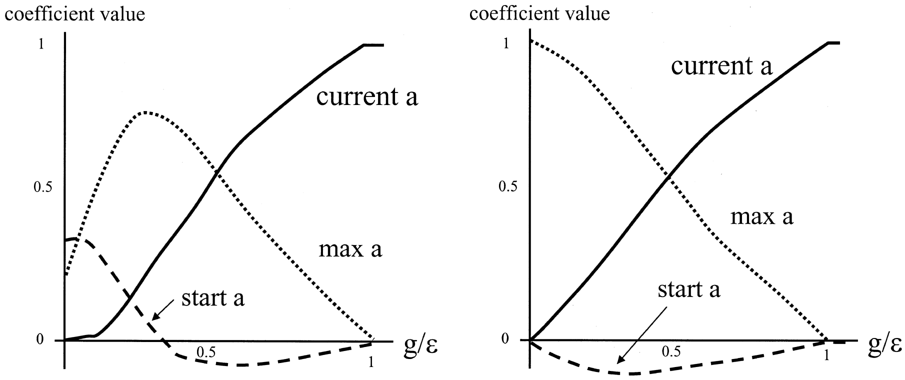

FIGURE 1. Regression results on simulated data: no discrimination (left panel) and the BDN-discrimination model (right panel).

the random walk, and $g/\varepsilon = 0$ implies wages are held constant when possible. For $g/\varepsilon \geq 1$, $w_t = a_t$, for all t—the wage coincides with the outside wage—so that a_t has a coefficient of unity and explains all the variation in w_t. However, over a substantial range of g/ε, the coefficient on max a dominates those on either current or start of job a. Interpreting max a as a measure of the tightest labour market (i.e., corresponding to minimum unemployment in BDN), this appears to be broadly consistent with the regression results in BDN and subsequent work.

We also simulated an asymmetric discounting version of the BDN model in this environment (again taking an an eight-year tenure and regressing w_t on the same variables as before). Asymmetric discounting allows the unconstrained wage change to be non zero (and thus is comparable with our model in which wage change equals δ): $w_{t+1}/w_t = \beta_w/\beta_f$ where β_w and β_f are the discount factors of workers and the firm, respectively, and we assume that $\beta_w \leq \beta_f$. This simply reflects the desire to shift the consumption of the more impatient party forward. In this case we define $g \equiv -\log \beta_w/\beta_f$. Results are in the right panel. The case $\beta_w = \beta_f$ considered by BDN corresponds to $g/\varepsilon = 0$, and only max a has non-zero coefficient; this was the main prediction of their model. However, the other coefficients are non zero at higher g/ε. In general, estimated coefficients are broadly similar to our model other than at low g/ε, again suggesting that the models have similar properties with respect to the standard regression equation.

3.2. Model Differences

While we have found that our model implies, like BDN, that minimum unemployment during a worker's tenure is likely to be an important determinant of his/her wage, and there are other similarities in predictions, there are also a number of important differences. We discuss both here. Looking at similarities first, the updating rule (6) that characterises the evolution of wages has a similar form in both BDN's mobility model and our equal treatment model; if we allow for asymmetric discounting: $w_{t+1} = \max\{\delta(\beta_w/\beta_f)w_t, \underline{w}_s\}$ in our model and the equation is the same in BDN except for the omission of δ (of course, $\underline{w}_s$ is solved for differently in the two models, but it has the same interpretation in BDN: it is the optimal current wage to pay to a worker in state s if she is getting from the future contract exactly her outside option). In the absence of further restriction that allow identification of discount factors, the updating equations are equivalent.[7]

There are however substantive differences between the two models. Looking at an individual worker, the initial wage (i.e., on joining the firm) is determined differently. In BDN's model, at the point of joining, the wage, say $w_\tau^{(\tau)}$, where τ is the entry date, is determined by auxiliary assumptions concerning the manner of competition in the labour market. They assumed that there is a perfectly elastic supply of identical firms, so that workers can extract all the available surplus from risk-sharing; combined with their assumption that workers can costlessly switch between firms, this implies that $w_\tau^{(\tau)} = \underline{w}_{s_\tau}$ (where again $\underline{w}_{s_\tau}$ is solved in BDN's model) since the outside utility offered by other firms will imply zero profits for these (identical) firms.[8] Consequently in BDN, even generalized to asymmetric discounting, a worker's wage depends only upon labour market states occurring during the current contract. By contrast, in our model, the wage at time of entry τ and subsequently may be determined by states prior to entry. At the point of joining, a worker may well receive a discounted utility above her outside option—this is likely if recent labour markets were tighter than the current one, so that risk-sharing considerations imply that incumbents are insulated from the fall in outside options. Thus a possible test to discriminate between the two models would be to see if labour market states before a worker's current tenure can help predict the current wage.

A further substantial distinction is of course that our model predicts that wages for all similar workers in a firm should follow the same path; by contrast in BDN this only happens when the participation constraint is binding for all workers. In a very tight market, then, BDN would predict similar wages, but, in a slack market recent hires will be paid less than those hired in better times, whereas in a slack market our model predicts recent hires are paid the same.

7. In either model a negative desired wage change might be considered implausible. With asymmetric discounting, however, this can be positive, and so long as productivity growth is large, none of the above conclusions will change.

8. See Sigouin (2004) for an alternative approach to determination of the initial wage in a similar model.

4. Conclusions

We find that a model with equal treatment (no "cohort effects") and full commitment on the part of workers, makes similar predictions concerning wage dynamics as the BDN model where workers can costlessly leave a firm. Simulations of an example suggest that our model predicts, as does BDN, that the tightest labour market during a worker's tenure may be a significant predictor of current wages, consistent with a growing body of empirical research. Nevertheless, the models differ in other predictions, and it will be a task of future work to devise tests to discriminate between them.

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