Web Appendix for "Redistributive Taxation and Personal Bankruptcy in US States"*

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Abstract

This Web Appendix contains estimation results for unsecured debt, further robustness checks for Tables 6 and 7 in the paper and a more formal treatment of arguments made in section 2 of the paper.

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1 Estimation results for unsecured debt

From 1988, households in their second and fifth interview of the CEX were asked separate questions about debts held in revolving credit accounts (including store, gasoline, and general purpose credit cards), in installment credit accounts, credit at banks or savings and loan companies, in credit unions, at finance companies, unpaid medical bills, and other credit sources. We sum these different items (adding negative balances held in checking and brokerage accounts) to construct gross unsecured debt for each household.[1] Excluded from the total are mortgage, and other secured debts. Thus, the debts analyzed differ from those in Gropp *et al.* (1997) which is important since the impact of bankruptcy exemptions on secured and unsecured debt is very different (Berkowitz and Hynes, 1999). The creditor has an additional claim to mortgage (and other secured) debt and can always recover the house (or other security) if the debtor defaults. The housing, or other exemptions will not affect the creditors rights in this case and hence we exclude such debts in the analysis.

As consumption, debt has been deflated with the consumer price index and is denominated in 1984 dollars. The mean level of debt in the survey is \$2,151. The median is \$331, while the 75th percentile is \$2,211 (similar numbers are reported by Cox and Jappelli, 1993, for the Survey of Consumer Finances).

Table A1 contains regression results for the relationship between bankruptcy exemptions and tax/transfer redistribution and the level of unsecured household debt. This regression is similar to previous research by Gropp *et al.* who use the 1983 wave of the Survey of Consumer Finances (SCF). This allows us to check how our new measures of bankruptcy exemptions and of redistributive taxes and transfers perform and gauge the robustness of the previous findings for our different data sample period 1988-2003.

Table A1 shows the estimates for the association of government redistribution through taxes and transfers and of the bankruptcy exemptions with unsecured debt. Debts b and the exemption x are measured in logs, or rather as $\log(1+b)$ and $\log(1+x)$. Column (1) uses the income compression measure while column (2) uses the mean marginal tax rates

[1] We concentrate on gross rather than net unsecured debt because this is the amount that is not repaid in bankruptcy.

as a regressor. We include and report estimates for a full set of household characteristics which control for observable heterogeneity that relates to permanent income and life-cycle circumstances and tastes among other things, and a set of state, year, and month dummies.[2]

Since households cannot report negative debts - such households would report zero debts - the estimation must solve this censoring problem. Rather than using a tobit regression, which imposes that the errors are normally distributed, we estimate the effect of taxes/transfers and of exemptions by a censored least absolute deviation (CLAD) regression as proposed by Powell (1984).[3] This semi-parametric estimator only imposes the weaker assumption that the error term in the latent regression is symmetrically distributed, and consistently estimates the median effect.[4]

The main focus of the analysis are the coefficient estimates of the bankruptcy exemption and of the tax and transfer system. Since we have household level data, we can estimate different coefficients of the exemption for homeowners and renters. This allows us to at least partially condition on the assets that the household owns. Hence, each regression includes the exemption level, included separately for homeowners and non-homeowners, and a dummy for the unlimited homestead exemption for homeowners.[5] This is in the spirit of Gropp *et al.* (1997) who have shown that the effect of the bankruptcy exemptions are different for high-asset and for low-asset households.

The coefficient estimate for redistribution through taxes and transfers is positive but not significant for either measure of the tax and transfer system in columns (1) and (2). Although insignificant, the positive sign is surprising: if taxes and transfers reduce intertemporal inequality, then increasing the amount of redistribution of the tax system should lower the need to borrow and save to smooth consumption over time.

[2]Since we include an age polynomial and a set of year dummies, this precludes using dummies for year of birth because they are not linearly independent.

[3]Results for tobit regressions are qualitatively similar for most regressors and are available on request from the authors.

[4]The true sample errors of the estimates depend on the unknown density of the errors at the median. Hence we calculate the standard errors of the estimated coefficients by bootstrapping using 100 draws.

[5]The difference in the effect of the housing and non-housing exemption on debt held by homeowners was not significant in our estimations. These results, omitted for brevity, justify adding the housing and non-housing exemptions together.

The coefficient estimate for the bankruptcy exemption is negative for renters. However, this effect is small (the coefficient implies that a 10% increase in the exemption reduces debts by 0.3% using the income compression measure, and 0.5% using the mean marginal tax rate) and is neither statistically nor economically significant. The estimated effect on those households who own their house is positive, much larger in size and significant at the 5% level. The coefficient suggests that unsecured debts increase by over 2% when the bankruptcy exemption increases by 10%. These results are qualitatively similar to those obtained by Gropp *et al.* (1997) who found that the bankruptcy exemptions had a negative effect on borrowing of low-asset households and a positive effect on high-asset households (although, in contrast to Gropp *et al.*, we only assign the housing exemption to homeowners in constructing our exemption measure). The results in our regressions show that the homestead exemptions have a significant positive effect on the level of household debt, suggesting that more generous bankruptcy exemptions can help households to smooth consumption.

The estimated coefficient of the dummy for the unlimited homestead exemption is also positive and significant in the regression at the 5% level. The estimate implies that households have over 50% more debt if they own their own home and live in a state in which the home is fully exempt from seizure should they declare bankruptcy. However, controlling for state fixed effects, as done in the regression, means that the coefficient of the unlimited homestead exemption dummy is identified only due to Minnesota replacing its unlimited homestead exemption by a maximum homestead exemption of $200,000 in 1993. Hence, the coefficient of the dummy is not well identified.

The coefficients of the household characteristics are very similar for both measures of the tax and transfer system. Table A1 reports that younger households (those around 30) have more unsecured debt and that debts decline steadily with age. This is consistent with standard life-cycle models of consumer behavior in which income increases over the working life. The table also shows that better educated households have more debt. This seems reasonable since these households have higher levels of permanent income relative to current income which they might want to bring forward at the early stage of their life-cycle. However, households where the head has completed a full college degree have less unsecured debt than if the head has only had some college education. Black and female households have lower levels of unsecured debt, as do married couples. Family size is positively associated with the level of debt, but family size squared has a negative coefficient. A similar pattern is apparent for income, and our results show that debts increase with income over the range of households in our survey. The regression also includes the real risk-free municipal bond rate as a proxy for the interest rate (we do not observe the interest rate in the debt contract). The coefficient is negative but statistically insignificant.

Dependent variable: Unsecured debt	(1)	(2)
income compression	0.381	-
	(0.903)	
mean marginal tax rate	-	0.442
		(2.523)
exemption × renter	-0.028	-0.048
	(0.284)	(0.305)
exemption × homeowner	0.213	0.214
	(0.054)	(0.061)
unlimited homestead exemption	0.576	0.559
	(0.190)	(0.178)
age/10	-6.404	-6.565
	(3.231)	(3.002)
age/10-squared	1.616	1.651
	(0.753)	(0.709)
age/10-cubed	-0.136	-0.139
	(0.057)	(0.054)
finished school	0.956	0.943
	(0.153)	(0.149)
some college	1.259	1.244
	(0.153)	(0.154)
full college degree	0.761	0.739
	(0.158)	(0.154)
black	-0.574	-0.577
	(0.126)	(0.116)
female	-0.208	-0.210
	(0.068)	(0.062)
couple	-0.228	-0.219
	(0.222)	(0.236)
ln(family-size)	1.092	1.076
	(0.379)	(0.400)
ln(family-size)-squared	-0.404	-0.396
	(0.168)	(0.173)
ln(income)	27.153	27.033
	(2.943)	(3.266)
ln(income)-squared	-1.287	-1.281
	(0.141)	(0.157)
interest rate	-0.062	-0.055
	(0.323)	(0.296)

Estimated by Censored Least Absolute Deviation (CLAD), with bootstrapped standard errors, in parentheses, using 100 repetitions. Regression included all households in the 18 largest states whose head was between 30 and 60 years old. Month, year and state dummies are included. Unsecured debts and the bankruptcy exemptions are measured in logs. 'Unlimited homestead exemption' is a dummy with the value one if there is no upper limit to the value of the housing exemption. The interest rate is the real municipal bond rate. The sample size is 34,085.

Table A1: Taxes/transfers, bankruptcy exemptions and unsecured debt

2 Further robustness checks

Tables A2 and A3 display results for the specifications estimated in Table 6 and 7 of the paper where we now use the mean marginal tax rate instead of the income-compression measure.

Column (1) in Table A2 contains the results for the OLS regression of bankruptcy exemptions on the mean marginal tax rate. Columns (2) to (4) display the results if we instrument the mean marginal tax rate by its two-period lag or the political variables, respectively.

Column (1) in Table A3 contains the results for the OLS regression of consumption inequality on bankruptcy exemptions and mean marginal tax rates. Columns (2) to (4) display estimates if we use consumption-growth inequality as dependent variable, where column (2) contains the OLS estimates and columns (3) and (4) the IV estimates.

Comparison of Table A2 with Table 6 in the paper and Table A3 with Table 7 in the paper shows that our results are robust to using mean marginal tax rates instead of the income-compression measure.

Dependent variable: Bankruptcy exemptions				
	(1)	(2)	(3)	(4)
mean marginal tax rate	-0.171**	-0.236**	-0.201**	-0.174*
	(0.020)	(0.028)	(0.029)	(0.013)
constant	0.093**	0.109**	0.099**	0.092**
	(0.006)	(0.007)	(0.007)	(0.004)
IV	-	lag	pol1	pol2
$Rank - test$	-	79.4	7.16	55.7
$(prob)$	-	(0.000)	(0.000)	(0.000)
$Sargan$	-	-	4.290	27.99
$(prob)$	-	-	(0.038)	(0.000)
N	420	384	358	358
R^2	0.777	-	-	-

Standard errors in parentheses allow for clustering by state. IV refers to whether the mean marginal tax rate is instrumented by the two-period lag (denoted 'lag') or by a set of political instruments: 'pol1' denotes the regression in which we use only the political affiliation of the state legislature and the tax efficiency index as instruments, while 'pol2' denotes the regression in which the full set of instruments is used. All regressions included a set of state dummies, and a dummy for unlimited homestead exemptions. Significance levels are denoted by $^+$ (10 percent), * (5 percent), and ** (1 percent).

Table A2: The relationship between mean marginal tax rates and bankruptcy exemptions

	(1)	(2)	(3)	(4)
Dependent variable:	$sd(c_{it})$	$sd(\Delta c_{it})$	$sd(\Delta c_{it})$	$sd(\Delta c_{it})$
mean marginal tax rate	-0.188**	-0.485**	-0.558**	-0.541**
	(0.070)	(0.117)	(0.138)	(0.135)
exemption	-0.042**	-0.052*	-0.050*	-0.049
	(0.014)	(0.023)	(0.023)	(0.034)
unlimited homestead dummy	-0.122*	-0.046	-0.012	-0.027
	(0.051)	(0.085)	(0.081)	(0.130)
constant	0.839**	0.665**	0.649**	0.634**
	(0.066)	(0.054)	(0.055)	(0.039)
IV	-	-	lag	pol
$Rank-test$	-	-	79.7	55.7
$(prob)$	-	-	(0.000)	(0.000)
$Sargan$	-	-	-	8.91
$(prob)$	-	-	-	(0.112)
N	420	412	384	358
R^2	0.153	0.108	-	-

Standard errors in parentheses allow for clustering by state. All regressions included a full set of state dummies. In columns (8) and (9) we instrument by the two-period lag (denoted ‘lag’) or by a set of political instruments (denoted ‘pol’). All regressions included a full set of state dummies. Significance levels are denoted by $^+$ (10 percent), * (5 percent), and ** (1 percent).

Table A3: The effect of mean marginal tax rates and bankruptcy exemptions on consumption insurance

3 A stylized model

In this section we construct a simple model with two periods labeled 1 and 2 in which the bankruptcy decision is modeled in a standard way (see, for example, White, 2006). The model motivates our empirical analysis. It shows that the effect of bankruptcy exemptions and redistributive taxes and transfers on consumer welfare is not as obvious as one might think, as both policies affect the bankruptcy decision and thus how banks price lending to households. We show under what conditions both policies are substitutes in providing partial insurance to consumers and illustrate the interactions between a linear redistributive tax/transfer scheme and a bankruptcy exemption. We choose a simple structure that allows us to derive some analytic results for a relatively general class of utility functions and probability distributions.

Basic setup. Agents are risk-averse and either borrow at interest rate r_2 from risk-neutral banks, or lend at the world risk-free interest rate r_f. The interest rate r_2 is endogenously determined and incorporates the bank's expectation about the agent's repayment behavior in period 2. Thus, the interest rate r_2 will depend on each agent's circumstances like his amount of debt (we drop the agent-specific index for convenience). In contrast, the world interest rate r_f is exogenous and constant. This assumption is common in the literature. For example, Athreya (2006) defends this assumption by noting that the ownership of wealth is fairly concentrated. Thus an exogenous interest rate can be motivated assuming a small group of agents which holds all assets, and is unaffected by bankruptcy procedures. Since the probability of default is weakly larger than zero, $r_2 \geq r_f$.

Agents are born in period 1 with endowment ω_1. We focus on a representative borrower whose choices are a function of current resources and of expected future endowment draws in period 2. This focus is justified since each US state's median household owes some unsecured debt.

Timing. Given the endowment ω_1 (normalized to one without loss of generality), agents decide how much to borrow and consume in the first period. They know that in period 2 they will receive an uncertain endowment

$$\omega_2 = \mu + \varepsilon_2 \ ,$$

where μ is known and ε_2 is random with mean zero. Agents expect their endowment to grow in period 2 if $\mu > 1$.

After the endowment draw in period 2, agents decide whether to declare bankruptcy and how much to consume. Given this setup, before bankruptcy in period 2 resources are defined as

$$\rho_2 = \begin{cases} \omega_2 - (1 + r_2)b_1 & \text{if borrow} \\ \omega_2 + (1 + r_f)a_1 & \text{if save.} \end{cases}$$

Depending on whether the agent saved a_1 or borrowed b_1 in the previous period, total resources are larger or smaller than the current endowment ω_2. In period 1, total resources, ρ_1, trivially equal the endowment ω_1.

Government policy. The government is responsible for bankruptcy law and for taxes and transfers. Agents are taxed or receive transfers depending on their level of resources ρ_t. We define ρ^+ so that agents with resources $\rho_t < \rho^+$ receive transfers whereas agents with resources $\rho_t > \rho^+$ are taxed. To make the model interesting we assume that government redistribution cannot be conditioned on assets or the agent's consumption/saving choice. Otherwise the distinction between redistributive taxes and transfers and resources redistributed because of bankruptcy filings would be arbitrary. In particular, we assume a tractable time-invariant linear tax/transfer schedule

$$\tau(\rho_t - \rho^+) \tag{1}$$

so that net resources are defined as $\widetilde{\rho}_t \equiv \rho_t - \tau(\rho_t - \rho^+)$.

This tax-schedule conveniently summarizes redistribution via the parameter τ. We focus on τ but it can be shown that shown that changing ρ^+ has similar effects. Moreover, we abstract from a more complicated reallocation of resources by taxes and transfer *across time* (think of a pension scheme) and focus entirely on the role of taxes and transfers for providing insurance in incomplete markets by compressing after-tax income distribution (Varian, 1980). It is this effect of taxes and transfers which is most related to insurance provided by consumer bankruptcy.

A larger τ not only implies higher marginal tax rates in good states of the world (for high draws of ε_t) but also larger transfers in bad states in which, as we will see below, the agent may decide to declare bankruptcy. Notice that the assets of agents are taxed, and debt and its interest can be deducted, as is realistic in the US for most of our sample period although tax reforms have implemented some changes (see Makin, 2001). For simplicity we assume that the same tax rate applies to the endowment and assets/debt. Moreover, we do not explicitly model the deadweight loss resulting from this policy. A deadweight loss could be added without adding further insight unless we let this loss depend on bankruptcy exemptions. Athreya and Simpson (2006) have looked at such interactions numerically. In their model, the moral hazard problem of unemployment insurance schemes is exacerbated by bankruptcy exemptions which shelter the consumption of agents especially from long-term shocks. Less unemployment insurance then increases search effort, reduces the unemployment rate, and thus also lowers default rates. Such possible interactions between the welfare system and consumer bankruptcy have already been recognized by Posner (1995). Although problems of hidden action are certainly important in the real world, we abstract from them here for simplicity. Whether bankruptcy exemption and redistributive taxation are substitutes already depends on certain conditions in our simpler framework so that additional complications would not add additional insights for the empirical application in the paper.

The second policy in the model is the bankruptcy exemption x, the level of resources that can be kept when the household defaults. Fay *et al.* (2002) have shown empirically that exemptions are important for consumers' bankruptcy decision in the US. We focus on this important policy variable since, while bankruptcy is regulated at the federal level, states are allowed to set their own level of exemptions. This is important since our empirical application exploits the variation in the exemptions over time within each state.

We make the simplifying assumption that exemptions apply to resources and not to positive assets, as in reality. This simplification is not important, however, since we assume, as is realistic, that the households can rearrange their portfolio and convert endowments into assets before declaring bankruptcy.

Bankruptcy decision. Agents declare bankruptcy in period 2 if they have borrowed and their total *after-tax* resources (if they repaid their debt) fall below the exemption level x,

$$\widetilde{\rho}_2 < x.$$

Note that we assume that the agent first pays taxes and receives transfers before making the bankruptcy decision which is realistic since US households cannot default on taxes. The critical level of resources *before tax*, below which the agent declares bankruptcy, is

$$\rho_2^* = \frac{x - \tau\rho^+}{1-\tau}.$$

Unsurprisingly, richer agents declare bankruptcy if the exemption level x is higher. In contrast, the effect of more redistribution (increasing τ) on the bankruptcy decision depends on whether agents are net tax payers or receive transfers at the exemption level x (whether ρ^+ is greater than or less than x). If agents with resources higher than the exemption level receive transfers, $\rho^+ > x$, then $\partial\rho_2^*/\partial\tau < 0$. In contrast, $\partial\rho_2^*/\partial\tau \geq 0$ if $\rho^+ \leq x$. For later reference note that the critical value in terms of endowments is given by

$$\omega_2^* = \frac{x - \tau\rho^+}{1-\tau} + (1+r_2)b_1. \tag{2}$$

Agents declare bankruptcy at higher endowment levels if they have to service more debt.

In the remaining part of this section, we first characterize the effect of taxes τ and the exemption x on expected utility in the second period for a given level of borrowing b_1. This allows us to explore how the two policies interact in providing insurance in the second period. We then analyze how the level of borrowing in period 1 depends on the two policies x and τ, for a given interest rate r_2. From this we learn how the two policies affect intertemporal smoothing motives. We are able to provide analytic results for a general class of utility functions and probability distributions. We then parameterize both the utility and probability function in order to characterize the equilibrium and optimal exemption level in period 1 (for endogenous b_1 *and* r_2).

Expected utility in the second period. Personal bankruptcy only matters in period 2 if agents have borrowed in period 1. Borrowing is optimal if the marginal utility in period 1, evaluated at net resources $\widetilde{\rho}_1$=$\widetilde{\omega}_1$, is larger than the expected marginal utility in period 2 *conditional on repayment* evaluated at the net endowment. (Note that endowments in the second period that are below ω_2^* do not enter the Euler equation because consumption is not affected by the borrowing decision for these endowments.) That is:

$$u'(\widetilde{\omega}_1) > \beta(1+r_2)\int_{\omega_2^*}^{\infty} u'(\widetilde{\omega}_2)f(\omega_2)d\omega_2, \tag{3}$$

where $u(.)$ is a strictly concave, continuous and differentiable utility function, primes denote derivatives, ω_2^* is the endowment below which the agent declares bankruptcy, β is the discount factor, r_2 is the interest rate at which the agent can borrow in period 1, and $f(.)$ is the probability density. Moreover, $\widetilde{\omega}_2 = \omega_2 - \tau(\omega_2 - \rho^+)$ is the net endowment in the second period if the agent has zero assets (no debt).

The expected utility of a borrower in period 2 is

$$\begin{aligned} u_2^b \;=\; & \int_{\omega_2^*}^{\infty} u((\underbrace{\omega_2 - (1+r_2)b_1}_{=\rho_2})(1-\tau) + \tau\rho^+)f(\omega_2)d\omega_2 \\ & + \int_{\rho_2^*}^{\omega_2^*} u(x)f(\omega_2)d\omega_2 \\ & + \int_0^{\rho_2^*} u(\omega_2(1-\tau) + \tau\rho^+)f(\omega_2)d\omega_2. \end{aligned} \tag{4}$$

The first line of expression (4) contains the utility of a borrowing agent if he repays in period 2. The second line is the utility if the bankruptcy exemption provides full consumption insurance. And the third line is the utility if the endowment in period 2 is so low that the bankruptcy exemption only provides partial insurance. Note that, as is realistic, agents who default on their debt cannot default on tax payments and can no longer tax deduct their debt and interest payments.

We now briefly mention how the interest rate is determined before we show how the insurance provided by bankruptcy is affected by the redistributive tax and transfer scheme.

Determination of the interest rate. A risk neutral bank in a competitive banking market sets the interest rate r_2 so that it receives the same expected return as lending on the world market at the risk free rate r_f. Clearly, the interest rate r_2 depends on the amount of debt outstanding since this influences the repayment probability (the bankruptcy threshold ω_2^* depends on the amount of debt b_1, see equation (2)). The arbitrage condition, which implicitly defines the interest rate r_2, is

$$\int_{\rho_2^* + \frac{C}{1-\tau}}^{\omega_2^*} (\omega_2 - x - C - \tau(\omega_2 - \rho^+))f(\omega_2)d\omega_2 + \int_{\omega_2^*}^{\infty} (1+r_2)b_1 f(\omega_2)d\omega_2 = b_1(1+r_f) \tag{5}$$

where C is the deadweight bankruptcy cost which prevents full insurance of risk-averse consumers by risk-neutral banks. This cost is borne by the bank, and reflects deadweight administrative and judicial costs. The first integral in the arbitrage equation is the expected repayment in the states of the world in which the agent partially defaults whereas the second integral is the expected repayment in those states where the agent fully repays. Note that we implicity assume that the bank does not incur the bankruptcy cost if the agent fully defaults. This assumption is not essential but is reasonable in our model since it is unclear why the bank would start costly procedures if it knows that it does not receive any net payment.

Policy substitutes? We now provide analytic results on how bankruptcy exemptions and redistributive taxes and transfers interact in this stylized model.

Analytical results. We show under what conditions bankruptcy exemptions and redistributive taxes and transfers are substitutes in terms of the expected utility in the second period.

Remark 1: *For given borrowing b_1, redistributive taxation lowers the welfare gains derived from a higher bankruptcy exemption:*

$$\frac{d\left(\frac{du_2^b}{dx}|_{b_1}\right)}{d\tau} < 0,$$

if consumers receive transfers at the bankruptcy threshold, consumers cannot tax-deduct their debt and some restrictions on the probability distribution are satisfied.

Proof: provided in the subsection of proofs below.

The intuition for this result is given in the subsection of the proofs. Next, we briefly show that compressing the distribution of net income reduces the desire to borrow for agents who expect higher gross income in the future. To make this point formally, we characterize the amount borrowed when the Euler equation

$$u'(\widetilde{\omega}_1 + b_1) = \beta(1 + r_2)\int_{\varepsilon_2^*}^{\infty} u'(\underbrace{\mu + \varepsilon_2 - (1 + r_2)b_1}_{=\rho_2} - \tau(\rho_2 - \rho^+))dF(\varepsilon_2) \quad (6)$$

is satisfied (assuming that the parameters are such that the agent finds it optimal to borrow). Recall that $\omega_2 = \mu + \varepsilon_2$ and that the amount borrowed is only repaid above the bankruptcy threshold ε_2^*. The bankruptcy threshold is defined as

$$\varepsilon_2^* = \frac{x - \tau\rho^+}{1 - \tau} + (1 + r_2)b_1 - \mu.$$

We show:

Remark 2: *For a given interest rate* r_2

- $db_1/dx|_{r_2} > 0$;
- $db_1/d\tau|_{r_2} < 0$ *if intertemporal resources are compressed (*$\rho^+ > \rho_1$*,* $\rho_2 > \rho^+$*) and all agents with resources less than the exemption level receive transfers (*$\rho^+ \geq x$*).*

Proof: provided in the subsection of proofs below.

The sign of the derivatives is intuitive. A higher exemption level x insures the agent in the bad states in period 2: he will repay the debt only for relatively high endowment realizations when the cost of repayment in marginal-utility terms is smaller. As is well known, this makes borrowing more attractive (see, for example White, 2006).

Instead, taxation in the good states of the world in which the agent repays increases the marginal-utility cost of repayment in the second period; and transfers in the first period lower marginal utility. Both effects make borrowing less attractive. Furthermore, if a larger τ decreases the bankruptcy threshold ε_2^*, for $\rho^+ > x$, debt is repaid in states with higher marginal utility which makes borrowing less attractive.

Intuitively, if redistribution through taxes and transfers decreases intertemporal inequality, the desire to borrow falls. This lowers the welfare gains derived from the exemption x. Formally, the interval in which the bankruptcy exemption provides insurance in the second period depends positively on b_1. In the extreme case in which taxes and subsidies completely align the marginal utility of present consumption with the discounted expected marginal utility of future consumption, agents do not borrow and the bankruptcy exemption is useless. We now provide a numerical example on the policy interaction allowing for borrowing b_1 and the interest rate r_2 to be jointly determined.

Numerical example. We have shown under what conditions redistributive taxation and bankruptcy exemptions are substitutes. However, for the derivations on insurance in period 2 we have conditioned on the amount borrowed b_1 whereas for the derivations on borrowing in period 1 we have conditioned on the interest rate r_2. With both b_1 and r_2 endogenous, an interpretable analytic solution is no longer obtainable unless strong assumptions are imposed on the utility function (such as constant absolute risk aversion). This section illustrates numerically the equilibrium in period 1 for a constant relative risk aversion utility:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}. \tag{7}$$

Moreover, we compute the indirect utility of a borrowing agent as a function of the policy parameters in order to show how the optimal exemption level depends on redistributive taxes and transfers.

The numerical algorithm to solve for the equilibrium in period 1 is simple: for given starting values for b_1 and r_2, we use the Euler equation (6) to iterate for the optimal b_1. We then update the bankruptcy threshold ε_2^* and use the bank's arbitrage condition (5) to solve for r_2. For the new values of b_1 and r_2, we restart the algorithm until convergence.

For illustration purposes we choose an exemption level x of 90% of first-period resources, which is in the range of plausible values for US states. This exemption level implies reasonable default rates and is substantially lower than the mean of second period endowment which is 1.4. We assume that $\rho^+ = 0.9 = x$ so that the comparative statics for taxes τ in Remark 2 apply and set the marginal tax rate $\tau = 0.2$ which equals the mean (federal plus state) marginal tax rate in Texas. We assume a rather small bankruptcy cost C of 1.5% in terms of first-period resources. Finally, the coefficient of relative risk aversion, $\sigma = 2$, is well in the range of commonly used values. The parameters are chosen in order to illustrate the main qualitative insights of our stylized model for borrowing agents. With the chosen parameters, borrowing is optimal (the agent is impatient and expects his income to rise 40% in the second period).

Table A4 summarizes the equilibrium values for the benchmark parametrization in column (1). Columns (2) and (3) display the results when we change taxes or the exemption level. Since the first period endowment $\omega_1 = 1$, borrowing is expressed as a fraction of it. In the benchmark case, the agent borrows an amount equivalent to 19% of these resources and defaults on the debt with a probability of 0.014, which is close to empirically observed frequencies. Although the default rate is small, the interest rate on borrowing is 0.032, 60% higher than the risk-free rate r_f.

In column (2) of Table A4, we increase marginal taxes/transfers to $\tau = 0.25$, approximately the mean marginal tax rate in US states like Maryland or Minnesota. In our numeri-

Variables	*Benchmark*	$\tau = 0.25$	$x = 0.85$
	(1)	**(2)**	**(3)**
borrowing b_1	0.1854	0.179	0.186
interest rate r_2	0.0321	0.0325	0.025
default prob.	0.0137	0.012	0.005

In the benchmark case, the risk-free interest rate $r_f = 0.02$, $\omega_1 = 1$, the bankruptcy cost $C = 0.015$, the bankruptcy exemption $x = 0.9$, the coefficient of risk-aversion $\sigma = 2$, the discount rate $\beta = (1 + 0.1)^{-1}$, $\mu = 1.4$, the marginal tax rate $\tau = 0.2$ while the temporary income shock $\varepsilon \sim N(0, 0.1 * \omega_2)$.

Table A4: Equilibrium values of borrowing b_1, interest rate r_2, and default probability

cal example there are two effects. More redistribution through transfers/taxes decreases the amount borrowed (as pointed out in Remark 2) and thus the default probability, but agents are taxed for some endowments where they partially default. Thus the bank recovers less in the case of default and overall the interest rate increases slightly in this example, despite the lower frequency of default.

Column (3) shows that reducing the exemption level to $x = 0.85$ leaves the amount borrowed nearly unchanged and does not decrease it as in the comparative statics of Remark 2. This happens because the lower exemption decreases the probability of default by so much that the interest rate falls substantially. The resulting wealth, income and substitution effects make borrowing more attractive and cancel the direct negative effect on borrowing.

After having characterized the equilibrium numerically, we are interested in computing the optimal exemption level based on the indirect utility function of the borrower in period 1. We consider the benchmark case with $\tau = 0.2$ and the case of higher marginal taxes/transfers $\tau = 0.25$. We find that higher marginal taxes/transfers decrease the optimal exemption level from $x^* = 0.871$ to $x^* = 0.845$. The implied elasticity is -0.12. This confirms the conditional results of Remarks 1 and 2 which showed qualitatively how redistribution through the tax system can lower the welfare gains derived from bankruptcy exemption by decreasing borrowing and insurance provided in the second period.

3.1 Proofs of the Remarks

We first provide some further results on how the interest rate depends on the exemption level and taxes and transfers. These results will be useful for the proof of Remark 1.

I. Further derivations on the interest rate

Total differentiating equation (5), plugging in ω_2^* and rearranging, we get

$$\begin{aligned} & \left((1-F(\omega_2^*))\left(b_1+(1+r_2)\frac{\partial b_1}{\partial r_2}\right)-C\frac{\partial \omega_2^*}{\partial r_2}f(\omega_2^*)-\frac{\partial b_1}{\partial r_2}(1+r_f)\right)dr_2 \\ & +\left((1-F(\omega_2^*))\left(\frac{\partial b_1}{\partial x}\right)-\left(F(\omega_2^*)-F(\rho_2^*+\frac{C}{1-\tau})\right)-C\frac{\partial \omega_2^*}{\partial x}f(\omega_2^*)-\frac{\partial b_1}{\partial x}(1+r_f)\right)dx \\ = & \ 0. \end{aligned}$$

Noting that

$$\frac{\partial \omega_2^*}{\partial x}=1/(1-\tau)+(1+r_2)\frac{\partial b_1}{\partial x}$$

and

$$\frac{\partial \omega_2^*}{\partial r_2}=b_1+(1+r_2)\frac{\partial b_1}{\partial r_2},$$

we find that for a given level of borrowing:

$$\left.\frac{dr_2}{dx}\right|_{b_1}=\frac{F(\omega_2^*)-F(\rho_2^*+\frac{C}{1-\tau})+\frac{C}{1-\tau}f(\omega_2^*)}{(1-F(\omega_2^*))\,b_1-Cf(\omega_2^*)b_1}>0\ , \tag{8}$$

where $F(.)$ is the cumulative distribution function. The intuition is that a higher exemption level x makes the agent default in more states of the world (recall equation (2)). This increases the interest rate which reflects the higher risk of default. If there is no deadweight loss, so that $C=0$, the size of the effect depends positively on the ratio of the probability of bankruptcy with partial default, $F(\omega_2^*)-F(\rho_2^*)$, over the probability of full repayment $1-F(\omega_2^*)$. Notice that the interest rate is only affected by the bankruptcy exemption through those states of nature in which the household repays some, but not all, of its debts. Only for these states can the exemption reduce the repaid amount, and states with full default are not relevant. For $C>0$, $dr_2/dx|_{b_1}$ increases since the bankruptcy cost is borne by the bank. Furthermore, we can show the following:

Remark A.1: *For a given level of borrowing b_1 and negligible bankruptcy cost ($C = 0$), a higher tax/transfer τ increases the costliness of the exemption in terms of larger interest payments:*

$$\frac{d\left(\frac{dr_2}{dx}|_{b_1} b_1\right)}{d\tau} > 0 \tag{9}$$

iff

$$\frac{f(\rho_2^*)}{1-F(\rho_2^*)} > \frac{f(\omega_2^*)}{1-F(\omega_2^*)}\left|\frac{\partial\omega_2^*/\partial\tau}{\partial\rho_2^*/\partial\tau}\right| .$$

If $|\partial\omega_2^*/\partial\tau| > |\partial\rho_2^*/\partial\tau|$ a necessary condition is that the probability distribution has decreasing hazard on the interval $(\rho_2^*;\omega_2^*)$. Otherwise decreasing hazard is a sufficient condition.

Proof: Define $\omega_2^+ \equiv \rho_2^* + C/(1-\tau)$. Then

$$\frac{d\left(\frac{dr_2}{dx}|_{b_1} b_1\right)}{d\tau} = \frac{\frac{\partial\omega_2^*}{\partial\tau} f(\omega_2^*) - \frac{\partial\omega_2^+}{\partial\tau} f(\omega_2^+) + \frac{C}{1-\tau}\frac{\partial\omega_2^*}{\partial\tau} f'(\omega_2^*) + \frac{C}{(1-\tau)^2} f(\omega_2^*)}{(1-F(\omega_2^*) - Cf(\omega_2^*))} - \frac{\left(F(\omega_2^*) - F(\omega_2^+) + \frac{C}{1-\tau} f(\omega_2^*)\right)\left(-\frac{\partial\omega_2^*}{\partial\tau} f(\omega_2^*) - C\frac{\partial\omega_2^*}{\partial\tau} f'(\omega_2^*)\right)}{(1-F(\omega_2^*) - Cf(\omega_2^*))^2}.$$

The sign of this derivative depends on the numerators which can be rearranged to

$$\begin{aligned}
&\frac{\partial\omega_2^*}{\partial\tau} f(\omega_2^*)\,(1-F(\rho_2^*+C)) \\
&-\frac{\partial\rho_2^*}{\partial\tau} f(\rho_2^*+C)\,(1-F(\omega_2^*)) \\
&+\xi(C)\ ,
\end{aligned}$$

where $\xi(C)$ contains all the other terms and $\xi(0) = 0$. Thus

$$\frac{d\left(\frac{dr_2}{dx}|_{b_1} b_1\right)}{d\tau} > 0$$

iff

$$\frac{\partial\omega_2^*}{\partial\tau}\frac{f(\omega_2^*)}{1-F(\omega_2^*)} > \frac{\partial\rho_2^*}{\partial\tau}\frac{f(\rho_2^*)}{1-F(\rho_2^*)}$$

and $C = 0$. If $x < \rho^+$ implies $\partial r_2/\partial\tau < 0$, then $\partial\omega_2^*/\partial\tau < \partial\rho_2^*/\partial\tau < 0$ (conditional on b_1). Then a necessary condition for the inequality above to hold is

$$\frac{f(\rho_2^*)}{1-F(\rho_2^*)} > \frac{f(\omega_2^*)}{1-F(\omega_2^*)}.$$

For $\partial r_2/\partial\tau > 0$ this inequality is even a sufficient condition. ■

The intuition for Remark A.1 is that, with increasing density and decreasing hazard, more redistribution increases the expected cost of bankruptcy in terms of larger interest payments (in the states of nature in which the agent repays). If agents receive transfers at resources smaller or equal than the bankruptcy threshold, the interval $(\rho_2^*\,;\,\omega_2^*)$ in which the bankruptcy exemption provides full insurance "shifts to the left". With decreasing hazard, this shift makes the interest rate more sensitive to changes in the exemption level because the relative probability mass associated with bankruptcy and partial default increases relative to the mass associated with repayment. Recall that in the interval of endowments with partial default the bank still receives some payment which decreases as exemptions become more generous. If $C > 0$, the condition to sign the derivative in Remark A.1 can no longer be interpreted in a straightforward way. As inspection of equation (8) suggests, for $C > 0$ the shape of the density also becomes important. Thus, parametric assumptions on the probability distribution in quantitative models are important for the size and sign of the policy interaction on the "cost-side".

II. Proof of Remark 1

II.A. Preliminaries We first provide further derivations which are useful. Totally differentiating (4) with respect to the exemption x we find for given b_1 that

$$\begin{aligned}\frac{du_2^b}{dx}|_{b_1} &= -b_1\frac{dr_2}{dx}|_{b_1}\int_{\omega_2^*}^{\infty} u'((1-\tau)\rho_2(\omega_2)+\tau\rho^+)f(\omega_2)d\omega_2 \\ &\quad + \left(\frac{\partial\omega_2^*}{\partial x}+\frac{\partial\rho_2^*}{\partial x}\right)(u(x)-u(x))\,f(\omega_2^*) \\ &\quad + (F(\omega_2^*)-F(\rho_2^*))\,u'(x)\end{aligned}$$

which simplifies to

$$\begin{aligned}\frac{du_2^b}{dx}|_{b_1} &= -(1-\tau)b_1\frac{dr_2}{dx}|_{b_1}\int_{\omega_2^*}^{\infty} u'((1-\tau)\rho_2(\omega_2)+\tau\rho^+)f(\omega_2)d\omega_2 \\ &\quad + (F(\omega_2^*)-F(\rho_2^*))\,u'(x).\end{aligned} \tag{10}$$

The first line of the derivative captures the cost of the bankruptcy exemption because of higher interest payments in the good states of the world. This effect is less important if much of the interest payment can be tax deducted. The second line contains the benefit of a higher exemption in the bad states in which bankruptcy provides full insurance. For $C = \tau = 0$, equation (8) implies that banks insure agents at an actuarially fair price and the sign of $du_2^b/dx|_{b_1}$ depends on the sign of

$$u'(x) - \frac{\int_{\omega_2^*}^{\infty} u'(\widetilde{\rho}_2(\omega_2)) f(\omega_2) d\omega_2}{\int_{\omega_2^*}^{\infty} f(\omega_2) d\omega_2}.$$

As in White (2006), this expression is positive for risk-averse borrowers with strictly concave utility since

$$\widetilde{\rho}_2(\omega_2) \geq x \text{ for } \omega_2 \in (\omega_2^*; \infty)$$

and thus

$$u'(x) > u'(\widetilde{\rho}_2(\omega_2)) \;\forall\; \omega_2 > \omega_2^*.$$

Thus, for $C = 0$, full exemption is optimal. Instead for $C > 0$ and $0 < \tau < 1$, insurance is actuarially unfair and the welfare gains from the exemptions are bounded. Nonetheless, unless bankruptcy costs are prohibitively high, some exemption will improve the welfare of borrowing agents by reducing consumption fluctuations.

II.B. Proof

Totally differentiating (10) for given b_1,

$$\begin{aligned}
\frac{d\left(\frac{du_2^b}{dx}|_{b_1}\right)}{d\tau} = & -\frac{d\left(\frac{dr_2}{dx}|_{b_1} b_1\right)}{d\tau}(1-\tau)\int_{\omega_2^*}^{\infty} u'(\widetilde{\rho}_2(\omega_2)) f(\omega_2) d\omega_2 \\
& + b_1 \frac{dr_2}{dx}|_{b_1} \int_{\omega_2^*}^{\infty} u'(\widetilde{\rho}_2(\omega_2)) f(\omega_2) d\omega_2 \\
& + \frac{dr_2}{dx}|_{b_1} b_1 \int_{\omega_2^*}^{\infty} (\rho_2(\omega_2) - \rho^+)\; u''(\widetilde{\rho}_2(\omega_2)) f(\omega_2) d\omega_2 \\
& + \frac{dr_2}{dx}|_{b_1} b_1 \; \frac{\partial \omega_2^*}{\partial \tau} u'(x) f(\omega_2^*) \\
& + \left(\frac{\partial \omega_2^*}{\partial \tau} f(\omega_2^*) - \frac{\partial \rho_2^*}{\partial \tau} f(\rho_2^*)\right) u'(x)\;.
\end{aligned}$$

where

$$\frac{\partial \omega_2^*}{\partial \tau} < \frac{\partial \rho_2^*}{\partial \tau} \leq 0 \ ,$$

if the agent receives transfers at the bankruptcy threshold. We assume that redistribution is such that agents pay taxes if they are able to repay their debt in full: $\rho_2(\omega_2) - \rho^+ > 0$, for $\omega_2 > \omega_2^*$.

Now recall that the sufficient condition of Remark A.1 implies for negligible bankruptcy cost ($C = 0$), that $d\left(\frac{dr_2}{dx}|_{b_1} b_1\right)/d\tau > 0$. If agents cannot tax deduct their debt then the second line of the derivative vanishes (which otherwise would be positive). Furthermore, given that $dr_2/dx|_{b_1} > 0$ and utility is strictly concave, $u''(\bullet) < 0$, we then know that the third and fourth line of the derivative are negative. If the density is increasing, $f(\omega_2^*) > f(\rho_2^*)$, then also the fifth line is negative so that the derivative can be unambiguously signed to be negative. ■

II.C. Some explanation

The interaction between redistribution and the bankruptcy exemption in period 2 can be decomposed into five different effects which correspond to the five lines of the derivative in the proof. The first four effects show how more government redistribution alters the cost of the bankruptcy exemption whereas the last effect captures changes in the benefits of the bankruptcy exemption:

(i) Larger transfers *increase the cost* of the bankruptcy exemption in terms of higher interest rates in the good states of the world, $\omega_2 \in (\omega_2^*; \infty)$. The sign of the effect follows from Remark A.1.

(ii) A higher marginal tax rate *decreases the cost* of the bankruptcy exemption in terms of higher interest rates if consumers can tax deduct interest payments on their debt in the good states of the world. This makes the bankruptcy exemption more attractive if the marginal tax rate is higher. In Remark 1 we rule out this policy complementarity as is realistic for most of the sample period in the empirical part (see Makin, 2001).

(iii) Larger transfers in the bad states of the world imply higher taxes in the good states of the world when debt is repaid (for $\rho_2 > \rho^+$ if $\omega_2 > \omega_2^*$). The higher interest payment resulting from the bankruptcy exemptions then becomes *more costly* in marginal-utility terms.

(iv) Larger transfers imply that agents who receive transfers declare bankruptcy only at a lower endowment ω_2 (the integration bounds shift). This increases the probability mass of states of the world in which the debt is repaid, and thus *increases the cost* of the exemption in terms of a larger expected debt burden.

(v) Transfers change the probability mass of the states of the world in which the exemption fully insures. If

$$f(\omega_2^*) - f(\rho_2^*) > 0$$

the probability mass decreases which makes exemption less attractive. Thus, for increasing density, more redistribution *decreases the benefits* of bankruptcy exemption.

III. Proof of Remark 2

Define net resources in the first period as $\widetilde{\rho_1} = \rho_1 - \tau(\rho_1 - \rho^+)$. Totally differentiating the Euler equation (6) we find

$$\frac{db_1}{dx}|_{r_2} = -\frac{\frac{\beta(1+r_2)}{1-\tau} u'(x)f(x)}{u''(\widetilde{\rho}_1 + b_1) + \beta(1+r_2)^2(1-\tau)\left\{\int_{\varepsilon_2^*}^{\infty} u''(\widetilde{\rho}_2)dF(\varepsilon_2) + u'(x)f(x)\right\}} > 0$$

$$\frac{db_1}{d\tau}|_{r_2} = -\frac{(\rho^+ - \rho_1)u''(\widetilde{\rho}_1 + b_1) + \beta(1+r_2)(\rho_2 - \rho^+)\left(\int_{\varepsilon_2^*}^{\infty} u''(\widetilde{\rho}_2)dF(\varepsilon_2) + \overbrace{\frac{\partial \varepsilon_2^*}{\partial \tau}}^{\leq 0 \ if \ x \leq \rho^+} u'(x)f(x)\right)}{u''(\widetilde{\rho}_1 + b_1) + \beta(1+r_2)^2(1-\tau)\left\{\int_{\varepsilon_2^*}^{\infty} u''(\widetilde{\rho}_2)dF(\varepsilon_2) + u'(x)f(x)\right\}}.$$

The Euler equation implies that b_1 is optimally chosen. Thus, the derivative of the Euler equation with respect to b_1 is negative for strictly concave utility functions. Therefore the denominator of the total derivatives is negative so that $db_1/dx|_{r_2} > 0$.

A larger τ is more likely to decrease b_1 if it compresses the resources in period 1 and period 2: $\rho^+ > \rho_1$ and $\rho_2 > \rho^+$; and certainly so if $\rho^+ \geq x$. ■

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