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Time domain performance of decimation filter architectures for high resolution sigma delta analogue to digital conversion

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ABSTRACT

We present the results of a comparison of different decimation architectures for high resolution sigma delta analogue to digital conversion in terms of passband, transition band performance, simulated signal to noise ratio, and computational cost. In particular, we focus on the comparison of time domain group delay response of different filter architectures including classic multistage FIR, cascaded integrator-comb (CIC) with FIR compensation filters, particularly multistage polyphase IIR filter, cascaded halfband minimum phase FIR filter, and multistage minimum phase FIR filter designs. The analysis shows that the multistage minimum phase FIR filter and multistage polyphase IIR filter are most promising for low group delay audio applications.

1. INTRODUCTION

In a sigma delta analogue to digital conversion ($\Delta\Sigma$ ADC) based high-resolution audio system, decimation filters are used for obtaining PCM data from density modulated 1-bit or multi-bit signals [1]. Most modern digital audio systems include some sort of oversampling and downsampling processes in either software format or integrated circuits.

Common practice in the audio industry is to use cascaded half-band linear phase FIR filters for interpolation or decimation processes. Recently, there has been increasing interest in adopting different filter architectures [2] to eliminate pre-ringing (mainly for DAC) and high group delay (for both ADC/DAC) caused by the linear phase design.

The advances in digital hardware fabrication now allow fairly silicon expensive structures to be used. The 64

times oversampling ratio audio band decimator can be easily implemented in silicon with a three stage FIR filter [3]. There are also commercial audio codec products that enable users to directly process modulator outputs or customize the internal digital filters.

In $\Delta\Sigma$ ADC/DAC for high-resolution audio systems, the significantly large number of taps and the multistage architecture introduce high group delay that may not be desirable for some live or low latency applications.

[4] showed the latencies of $\Delta\Sigma$ ADC/DAC, which are mainly contributed by the group delay of internal digital filters and can be as high as 1.5 milliseconds. Many live audio applications or electronic musical instruments and software synthesizers require overall latency less than a few milliseconds. In these situations, the phase response can be diffused by the live environments, and hence becomes less important. And the high group delay caused by linear FIR filters can be undesirable. Therefore, it would be interesting to see how low group delay filters perform in comparison with classic linear FIR filter within multiple constraints such as cost, signal to noise ratio (SNR), and filter characteristics.

In this paper, we evaluated time domain performances of different decimation filter architectures with typical anti-aliasing filter design specifications for the $\Delta\Sigma$ ADC as in [3]. The tradeoffs of filter characteristics are discussed as well.

2. BASIC CONCEPT OF DECIMATION FILTER

The principle of the decimation process is similar to sample rate conversion, for which the Nyquist theorem has to comply to avoid the aliasing. Decimation can be treated as two cascaded function blocks: the downsampling process and the anti-aliasing filtering process. To downsample an input signal $x(n)$ with positive integer factor M , the output signal can be represented as $y(m)=x(Mn)$. If there is any frequency component greater than $f_s/(2M)$ in the original signal, where the original sampling frequency is f_s , the downsampling process will result in aliasing. In order to avoid the aliasing problem, a low-pass filtering process $H(z)$ is needed in decimation [5].

The purpose of the decimation filter in $\Delta\Sigma$ ADC is threefold:

- To avoid the aliasing in the decimation process.

- Help relax the analogue anti-aliasing filter design requirements.
- To remove quantization noise caused by the $\Delta\Sigma$ modulator and to obtain the effective number of bits (ENOB) in PCM format.

Almost any type of lowpass filter design techniques can be used for to decimation filter design [1] [6]. However because the $\Delta\Sigma$ ADC has its own characteristics and specific application requirements, the filter design work has always been the tradeoff of various design and implementation constraints.

The straightforward design can be linear phase single stage FIR lowpass filter. The order of FIR filter N can be estimated by the Equation 1 as summarised in [1][6]:

$$N \approx ((\log_{10} \delta_s)[a_1(\log_{10} \delta_p)^2 + a_2(\log_{10} \delta_p) + a_3] + a_4(\log_{10} \delta_p)^2 + a_5(\log_{10} \delta_p) + a_6) f_s / \Delta f \quad (1)$$

Where δ_p is passband ripple, δ_s is stopband ripple in linear, $a_1=0.005309$, $a_2=0.07114$, $a_3=-0.4761$, $a_4=-0.00266$, $a_5=-0.5941$, and $a_6=-0.4278$. The Δf is the transition bandwidth and f_s is the sampling frequency at oversampled rate.

When the oversampling ratio is large and the desired transition bandwidth of decimation filter is narrow, the order N can be very large, i.e., up to several thousand [1][6]. So although a single stage FIR filter can be realized, it is sometimes impractical due to this extremely high order. A more effective approach is to use cascaded multistage design [7], which provides an efficient general solution for decimation, interpolation and narrow band filter design. [7] also indicates the duality of the decimation and interpolation processes, so the same filter structure can apply to both.

For decimation filters in $\Delta\Sigma$ ADC, significant effort has been made to use simplified filter structures and implementation methods [8] [9] [10] [11] of multistage design. Among these methods, two important approaches are the cascaded integrator-comb (CIC) filter structure [8], and using halfband or N-band filters [12][13]. The polyphase filter structures [14] have also been widely adopted as effective implementation in multirate signal processing, including decimation and interpolation.

3. GROUP DELAY OF THE DECIMATION FILTER IN $\Delta\Sigma$ ADC

One important measurement of time domain performance of a digital filter is group delay. The group delay of a digital filter is defined as the first derivative of phase response as in Equation 2,

$$D_M(\omega) = -d\phi(\omega)/d\omega, \quad (2)$$

where ϕ is the total phase shift in radians, and ω is the angular frequency in radians per unit time. When the phase is linear then the group delay is constant. For non-linear filters, the group delay is a function of frequency. The decimation filter is essentially a digital anti-aliasing filter. Therefore the filter is typically designed and normalized at input sampling rate. The group delay at the output sampling rate can be calculated as in Equation 3, where M is the decimation factor.

$$D(\omega) = \frac{D_M(\omega)}{M} \quad (3)$$

For linear phase FIR filter, the group delay is around half the filter order N . Hence higher order results in higher group delay. The multistage design significantly reduces the filter order in total. However due to the fact that the stages operate at decreasing sampling frequency, the overall group delay normally is worse than the single stage filter by the same filter design method.

$\Delta\Sigma$ ADC is commonly used in high resolution audio because it can achieve more than 20bit ENOB (Effective Number of Bits) [27]. The higher oversampling ratio of the $\Delta\Sigma$ modulator also helps improve the SNR as well as signal-to-noise-and-distortion ratio (SINAD). Therefore, a more restricted decimation filter specification is needed in this case in terms of good stopband attenuation, small passband ripples and narrow transition band in order to obtain the PCM signal with equivalent ENOB. A linear phase digital filter to meet such requirements normally has high order and a multistage design. But in both cases, it worsens the group delay response.

Therefore, it is well known that the group delay of the digital decimation filter is the largest contributor to the latency of $\Delta\Sigma$ ADC [4] [26]. Minimum phase FIR and IIR filters can be used in delay critical applications when phase linearity is not required. Although different filter architectures can be used as decimator, such as minimum phase FIR or IIR filter [1], to the best

knowledge of the authors, there is little literature available to provide detailed qualitative or quantitative reviews of how different decimation filter architectures impact time domain performance in high resolution audio $\Delta\Sigma$ ADC.

4. EVALUATION METHODOLOGY

When the linear phase in the passband is not restricted, the decimation filter can be any type of lowpass filter. The design space is so wide that there is no systematic approach to optimal design choice [1]. Therefore we have to properly consider the different filter architectures for evaluation with justified rationale.

4.1. Selection of testing filters

We evaluated the time domain performance of the following main filter architectures with the typical filter design specifications in Table 1, based on a commercial ADC product, as specified in section 3 of [3]. The traditional and modern linear phase filter as well as the nonlinear phase, low group delay filters were evaluated. All the filters evaluated should satisfy the 90 dB stopband attenuation specification. The group delays of filters are calculated and compared. The group delay response figures are also provided to assess the effects of group delay distortion of nonlinear filters.

Parameters	Desired values
Decimation Factor	64x
Output Sampling Rate	48 kHz
Input Sampling Rate	3.072 MHz
Stopband Attenuation	> 90 dB
Passband Ripple	< 0.006 dB
Passband edge	21.6 kHz
Stopband edge	26.4 kHz

Table 1 Filter Design Specifications

4.1.1. Linear phase single stage FIR and multistage FIR filters

The linear phase single stage FIR filter and the multistage FIR filter are well-understood decimation approaches [6][7]. They can be designed by Windowed-Sinc or optimal design methods, and they provide a good reference design in comparison with other architectures. The optimal design should give the minimum order of the filter. Hence it could help reduce

group delay. In multistage filter design, the number of stages can be optimized as well.

In this case, the following filters are investigated:

- A single stage FIR filter designed by windowed-sinc method with Kaiser Window (Kaiserwin). This design normally provides very good performance among different window functions.
- A popular optimal equiripple FIR filter in single stage for decimation.
- A 3-stage FIR filter designed by the optimal method.

4.1.2. Cascaded CIC filter with linear phase FIR compensation

The CIC filter [8] is a very cost effective filter structure without multipliers. It is widely used in decimation. CIC filters are inherently linear phase, hence with constant group delay. Due to its simple and regular representation, the design of the CIC filter has less control of fine tuned parameters such as passband ripple and transition bandwidth. Therefore compensation filters are always adopted to improve the passband and other performances.

4.1.3. Six-stage half-band FIR filters with linear phase

The halfband filter is another effective architecture [12][13] used in decimation. 64 times decimation can be realized by 6 cascaded halfband filters with each performing decimation by a factor of 2. This design [12][10][16] should have the theoretical minimum taps within the FIR decimators catalogue. However the additional stages may complicate the control structure and have negative impact on group delay. Therefore, this filter is designed for evaluating the group delay.

4.1.4. Multi-stage polyphase IIR filters

Compared with FIR filters, the same magnitude response can generally be achieved by IIR filters with less coefficients. The IIR filter also typically has less group delay but with phase distortion.

The FIR filter is commonly used in multirate signal processing due to the effective filter structure realizations, such as the polyphase network [14], and the linearity requirements in most applications. But when nonlinear phase is allowed, the research [17][18][19] shows that recursive filters can also be designed and

realized in a very cost effective way, especially halfband design with allpass polyphase decomposition.

In addition, the phase of a recursive filter can be equalized to approximate a linear phase filter. Thus, it would be interesting to find out how linear phase IIR filter performance in the time domain compares with linear phase FIR filters as well.

In this case we designed two types of IIR filters:

- the 6 cascaded halfband IIR filter with elliptic response.
- the 6 cascaded halfband IIR filter with quasi-linear phase response.

4.1.5. Multistage minimum phase FIR filters

Minimum phase FIR filters with all zeros within the unit circle should have theoretical minimum group delay, and hence the fastest signal response. In this case, we design two minimum phase multistage FIR filters based on two typical effective linear phase designs:

- 3 stage minimum phase FIR filter.
- 6 stage minimum phase halfband filter.

A summary of evaluated filters is given in Table 2.

Filter Type	Filter code
Kaiserwin FIR	Kaiser
Equiripple FIR	Eqrip
3-Stage Equiripple FIR	3-stage
3-Stage FIR minimum Phase	3-min
CIC without compensator	CIC
CIC with compensator	CICom
Six-stage halfband FIR	6hb
Six-stage halfband minimum phase FIR	6hbmin
Six stage elliptic IIR filter	6IIR
Six stage Quasi linear IIR filter	6IIRlin

Table 2 List of evaluated filters

4.2. The filter performances matrix

Although the filters are designed to meet the specifications in Table 1, the actual designed filter may result in slightly different performances in terms of magnitude responses. Therefore some comparisons of magnitude response are also presented to see the

correlation between the frequency domain and time domain performances.

The theoretical implementation cost is given in terms of the number of multipliers, the number of adders. The number of multiplications and additions per input sample for these filters will also be compared. The theoretical SNRs will be evaluated by using a Matlab Simulink model of the $\Delta\Sigma$ ADC with full amplitude sinusoid signals as inputs.

5. RESULTS AND DISCUSSIONS

In this section the evaluation results of different types of filters are presented. Firstly, the designs of different types of filters are discussed. The correlation between various aspects filter design and its group delay properties are explored. Then the summary of group delay of all evaluated filters is presented and discussed.

5.1. Filter design and group delay impact

5.1.1. Linear phase single stage and multistage FIR filters

Design Considerations

Two FIR filter design methods are used for design of single stage FIR filters. According to Equation (1) and the specification (Table 1), the filter order is estimated up to 2314. The single stage filter can be designed by the Kaiser Window (Kaiserwin) method with very good passband and stopband performance. The Kaiser Window design meets the design specifications but with overestimated filter order N . However, the optimal equiripple algorithms sometimes underestimates the order, which is close to but does not meet the specifications.

The multistage linear phase FIR filter design uses three stages by the equiripple method. The decimation factors are $/8$, $/2$, and $/4$ respectively. The three stage design correlates with the decimation architecture in AD1877, as described in [3]. The second stage has decimation factor of 2, which is also a halfband filter. The stage 2 filter coefficients have zeros in every second order except that of the central point. The number of stages is also regarded as optimal with the automatic design algorithm from Matlab.

Results and Discussions

Table 3 shows a comparison of the single stage Kaiserwin design, the equiripple design, and the three stage equiripple design in terms of filter order and magnitude responses. Figure 1 shows the passband ripple at -0.01 dB to 0.01 dB of three different designs. Figure 2 shows the group delay of three filters. They all have constant and relatively high group delays, which are in the range $500\mu\text{s}$ to $600\mu\text{s}$ delay at 48 kHz sampling frequency (for detailed group delay values see Table 4). The Kaiserwin filter has better passband performance than the other two but with highest filter order N . Hence it also has highest group delay. The 3 stage equiripple filter has fewer orders in total but it has higher group delay than the same equiripple filter with single stage. This shows that the multistage structure normally worsens the group delay response.

Filter	Order	Passband Ripple	Stopband attenuation
Kaiser	3658	0.0005 dB	91.34 dB
Eqrip	3023	0.0045 dB	89.95 dB
3-stage	39-14-193	0.0043 dB	90.34 dB

Table 3 Compare single stage FIR filters with multistage FIR filters

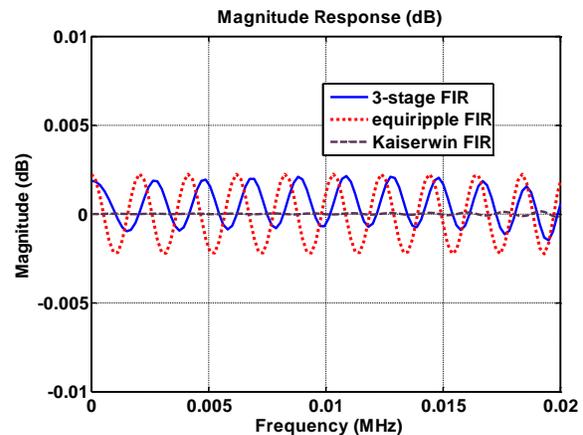


Figure 1 Magnitude passband of 3-stage FIR, equiripple FIR and Kaiserwin FIR filters

The 3-stage linear phase FIR is a typical implementation with the specified design criteria. Hence it will be used as reference design to be compared with other filter architectures.

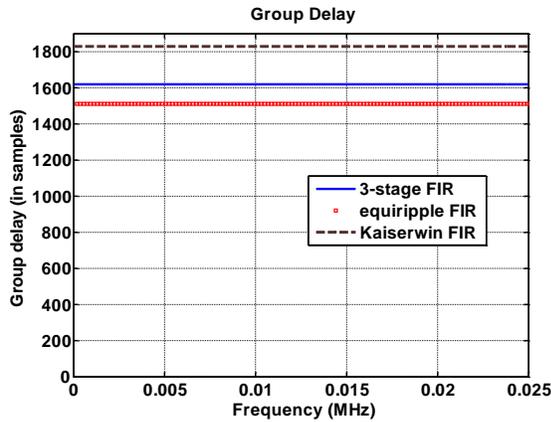


Figure 2 Magnitude passband of 3 stage FIR, equiripple FIR and Kaiserwin FIR filters

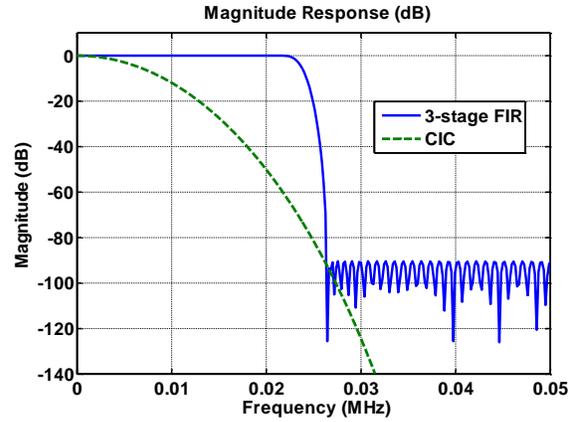


Figure 3 CIC without compensator in comparison with reference filter

5.1.2. Cascaded CIC filter with linear phase FIR compensation

Design Considerations

There are various compensation methods to improve CIC frequency responses since the initial CIC concept from Hogenauer in 1981. We are interested in time domain performance on a typical CIC filter with an FIR compensation. Therefore a CIC filter and a linear phase FIR compensator are designed.

Figure 3 shows the magnitude response of the CIC without compensator. It clearly shows the passband performance does not meet the specification in comparison with reference design.

We designed the FIR compensator to flatten the passband ripple within the design specification, as shown in Figure 4. But in this case the transition band is still not compensated well in this case.

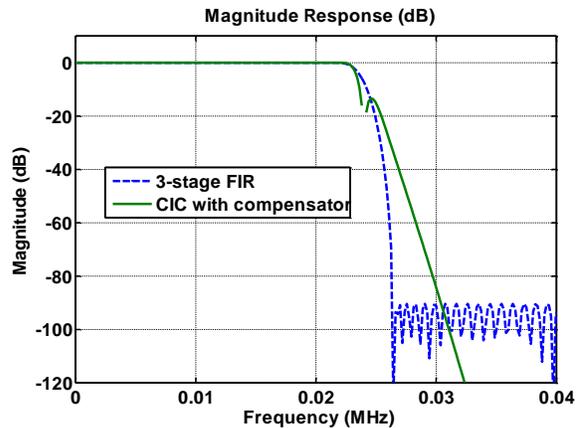


Figure 4 CIC Filter with compensator in comparison with reference design

Results and Discussions

Figure 5 shows the group delay of CIC and CIC with compensator in comparison with the reference design. CIC filters are inherently linear phase, hence with constant group delay. This CIC filter without compensation has 19 sections with constant group delay of 598.5 samples (Table 4) but with unsatisfactory passband performance. To flatten the passband within specification, a fairly expensive linear phase FIR filter design method is required. Therefore overall it illustrates high group delay. Our compensator design results in group delay of 4022.5 samples (Table 4). Reducing the sections will decrease group delay but with worse passband and transition band performance.

There are advanced CIC filter design and compensation methods. In general they are low pass filter design

techniques and thus they may not be very helpful in terms of reduction of group delays. The details of these methods are beyond the scope of this paper.

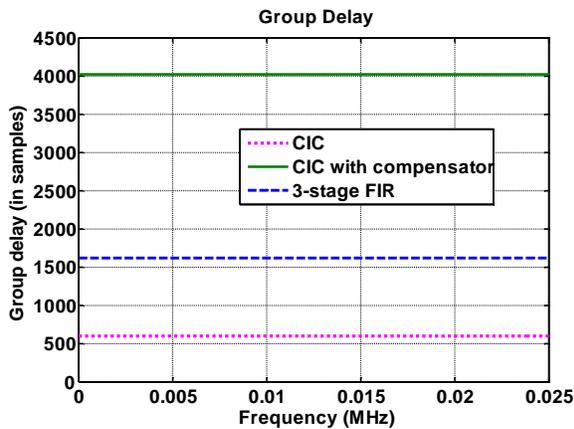


Figure 5 Group delay of CIC and CIC with compensator in comparison with reference design

5.1.3. Six stage linear phase halfband FIR filter

Design Considerations

Halfband is a class of N -band filters where the number of bands N is 2. It is an effective filter structure and design method for decimation. Halfband is most effective in N -band filter class in terms of filter coefficients. So the 64 times decimation filter can be designed by cascading six halfband filters.

In [10], the author designed a 64 times decimation filter with both cascaded CIC and FIR filters and 6 stage halfband FIR filters. We designed a similar 6 stage halfband filters to meet the specification defined in Table 1 to compare its time domain performance.

Results and Discussions

The 6 stage halfband FIR filter performs well in terms of magnitude response (see Figure 6). It has lower implementation cost (Table 5) than other FIR filter design methods, but it worsens the group delay as compared with the reference design (Figure 7) and other linear phase FIR filter design methods (Table 4).

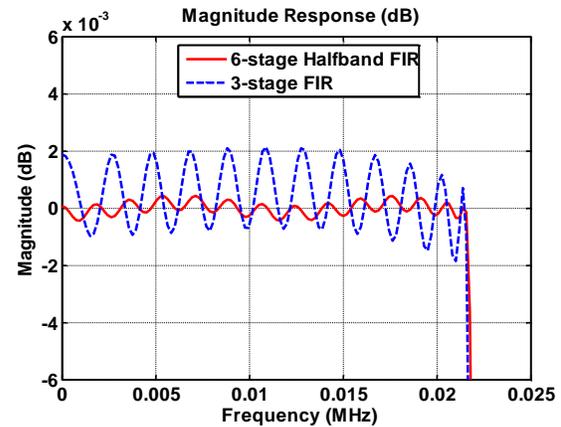


Figure 6 Passband performance of 6 stage halfband FIR filter in comparison with reference design

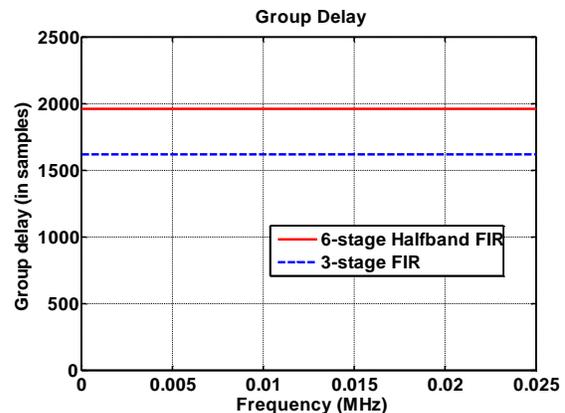


Figure 7 Group delay of 6 stage halfband FIR in comparison with reference design

5.1.4. Multi-stage polyphase IIR filters

Design Considerations

It is commonly believed that the IIR has less group delay but with non-linear phase response. The IIR filter can have an effective realization structure, which is suitable in decimation and multirate signal processing as well [20], especially by halfband design with allpass polyphase decomposition.

Since the technique is available to design linear phase (quasi-linear) IIR to take advantage of the efficiency of IIR filter while maintaining the linear phase. Therefore it would be interesting to see if the quasi-linear phase could help reduce group delay as well. In this case we designed two types of six stage IIR filters.

- The 6 stage quasi-linear IIR filter with quasi-linear phase on each stage.
- The 6 stage elliptic IIR filter with elliptic frequency response on each stage.

Results and Discussions

The IIR filters perform very well in passband and satisfy the stopband and transition band requirements. Also the design results in a very efficient theoretical implementation cost as shown in Table 5. Figure 8 illustrates the passband performance of IIR filters in comparison with the reference design.

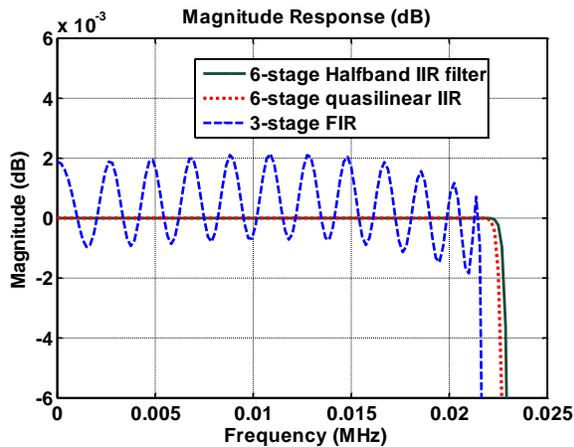


Figure 8 Passband performance of 6-stage elliptic IIR and 6-stage quasi-linear IIR decimator in comparison with reference design

Figure 9 shows that the quasi-linear IIR filter has almost constant group delay around 1514 samples at passband. It has slightly lower group delay than the reference design but still in similar scale.

The elliptic halfband IIR filter has very low group delay which is far better than the linear phase filters. However it has group delay distortion due to the nonlinearity of filter phase response. As shown in Figure 9, the group delay is 176.6 samples at frequency zero and 410.5 at frequency 20 kHz.

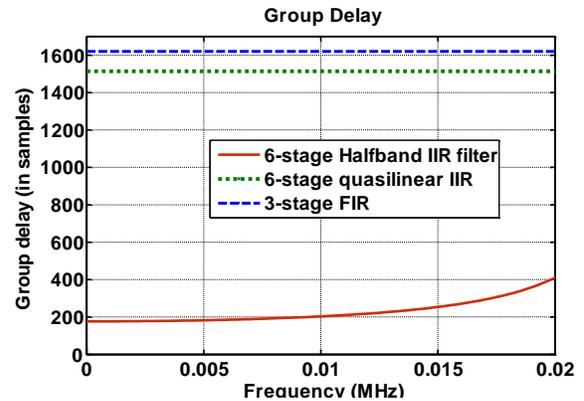


Figure 9 Group delay of 6 stage quasilinear IIR and 6 stage elliptic IIR in comparison with reference design

5.1.5. Multistage minimum phase FIR filters

Design Considerations

Theoretically, the minimum phase FIR filter has the fastest signal response as compared to equivalent nonminimum phase approaches. It would be interesting to see how minimum phase FIR filter performs in the decimation filter design. Based on two effective linear phase filter architectures: “6 stage halfband FIR filter” and “3 Stage FIR”, we designed the minimum phase version of these two architectures. We replaced each stage with a minimum phase FIR filter with the same magnitude responses by using a polynomial roots finding design algorithm. The minimum order of each stage might not be optimal (actually the optimal minimum phase FIR filter design algorithm has a convergence problem when the filter order is large). However, the algorithm we used has good numerical robustness and produces almost identical magnitude response as the linear phase version even for high order filters.

Figure 10 shows the comparison of impulse response (IR) of one stage of linear phase FIR filter and the impulse response of a minimum phase FIR filter which can produce exact magnitude response. The linear IR will be replaced by minimum phase IR in our design.

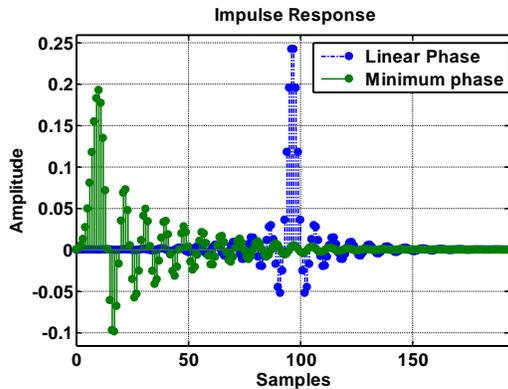


Figure 10 Impulse responses of minimum phase FIR and linear phase FIR filters

For the minimum phase 6 stage halfband design, each stage is a minimum phase halfband filter, which decimates the sampling frequency by 2. However in this case the “halfband” is in terms of the frequency response properties. The minimum phase halfband filters do not have the efficient coefficients property as linear phase halfband filters.

Results and Discussions

Both 3 stage minimum phase FIR filter and 6 stage minimum phase halfband FIR filter perform very well in time domain in comparison with the reference design, as shown in Figure 11. The group delay of the minimum phase 6 stage halfband FIR filter has similar shape as 3-stage minimum phase FIR filter with slightly higher delay (see detail in Table 4).

For 6 stage minimum phase halfband FIR filter, the group delay is 164.4 samples at frequency zero and 387 samples at frequency 20 kHz. For 3 stage minimum phase FIR filter, the group delay is 155 samples delay at frequency zero and 380 samples at frequency 20 kHz. Although group delay distortion happens in the 3 stage minimum phase FIR filter, within the audio band, this is equivalent to only 3.5 samples difference at the output sampling rate.

5.2. Summary of group delay of all evaluated filters

Table 4 shows the group delays of all the filters we evaluated with the equivalent delay time at output sampling rate. The 3 stage minimum phase FIR decimator, 6-stage minimum phase halfband FIR decimator, and 6 stage multistage IIR decimator perform very well in terms of low group delay. There

are some group delay distortions within the passband, as shown in Figure 12. There is a trend to high group delay near the Nyquist frequency. However it is only 3 to 4 samples difference in relation to output sampling rate.

Among these three low group delay decimators, the 3-stage minimum phase FIR filter has lowest group delay. The 6-stage halfband IIR filter has lowest theoretical implementation cost (Table 5) and the best passband performance (Figure 8). However there are complications for practical implementation since more stages normally requires a larger stage control structure.

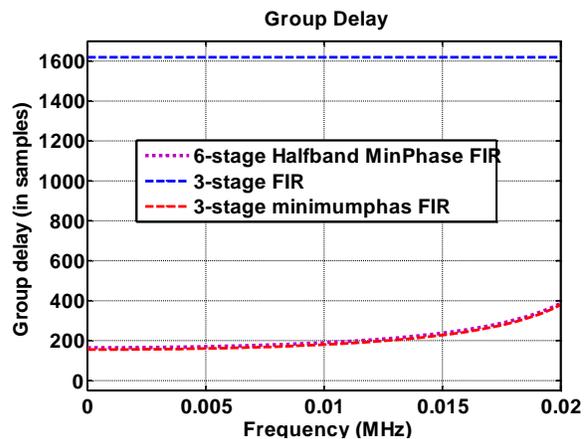


Figure 11 Group delay of 6 stage halfband FIR with minimum phase and 3 stage minimum phase FIR

Group delay for linear phase filters		
Filter	Group delay (samples)	Delay at 48 kHz (µs)
Kaiser	1829	595
Eqrip	1511.5	492
3-stage	1619.5	527
CIC	598.5	195
CICom	4022.5	1309
6hb	1961	638
6IIRlin	1514	493
Group Delay for nonlinear phase filters		
Filter	Group delay (samples)	Delay at 48 kHz (µs)
3-min	155 - 380	50 - 123
6hbmin	164.4 - 387	53 - 126
6IIR	176.6 - 410.5	57 - 134

Table 4 Group delay of different evaluated filters

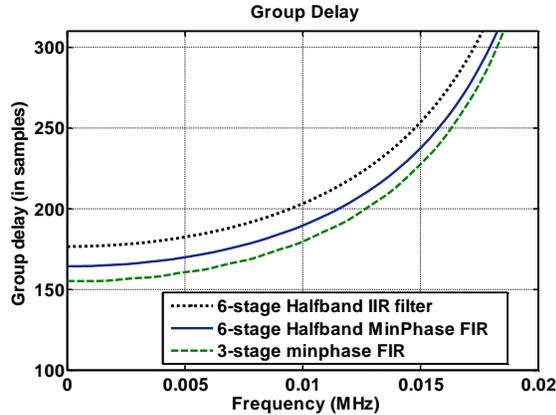


Figure 12 Group delay of 6-stage halfband FIR, 3 stage minimum phase FIR, and 6 stage halfband IIR filters

5.3. Compare Cost and SNR

Table 5 shows the theoretical implementation cost of ten filters evaluated. In the table the “NM” indicates “Number of Multipliers”, “NA” indicates “Number of Adders”, “M/I” indicates “Multiplications per Input Sample”, and “A/I” indicates “Additions per Input Sample”

Filter	NM	NA	M/I	A/I
Kaiser	3659	3658	57.1719	57.1562
Eqrip	3024	3023	47.25	47.2344
3-stage	243	240	8.5938	8.3906
3-min	249	246	8.9688	8.7656
CIC	1	38	1	19.2969
CICom	109	145	2.6875	20.9688
6hb	96	90	6.9531	5.9688
6hbmin	174	168	10.9531	9.9688
6IIR	19	38	1.6719	3.3438
6IIRlin	33	66	1.9062	3.8125

Table 5 Implementation cost of different filters

In order to verify whether the different decimation filter architectures affect the overall SNR of the ADC system, we converted the designed decimation filters into Matlab Simulink model blocks. The decimation block processes the simulated 1-bit first order $\Delta\Sigma$ modulator output, and outputs PCM data. The input signals are full amplitude sinusoid waveform with different frequencies, and the output data is calculated by FFT-based SNR estimation [21]. Table 6 shows the SNRs at three frequencies at typical low, mid and high audio band. It shows that there are no significant differences between different types of decimation filters.

Figure 13 shows the Matlab Simulink model of a decimation subsystem which consists of 6 cascaded

minimum phase halfband filters. Figure 14 shows the step response of three minimum phase decimators (the second to fourth display) in comparison with linear FIR decimator (at the top display). The one grid of X axis is simulated time of 0.5 millisecond. It shows linear decimator has around 0.5ms latency, whereas the minimum phase ones are shorter.

Simulated SNR for selected filters			
Filter	Frequency of Input signals		
	500 Hz	3000 Hz	12000 Hz
Eqrip	-120.5733	-107.411	-107.177
3-min	-120.6725	-107.3478	-107.1542
6hbmin	-120.8044	-107.3686	-107.1479
6IIR	-120.8078	-107.3693	-107.1469

Table 6 Simulated SNR values

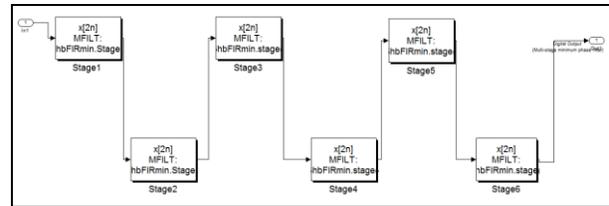


Figure 13 Simulink model for a subsystem of cascaded 6 stage halfband minimum phase filters

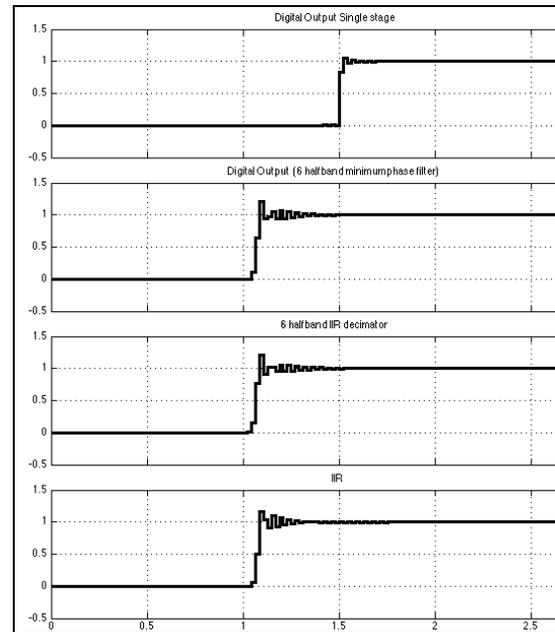


Figure 14 simulated step responses of Linear phase FIR, 3-stage minimum phase, 6-stage halfband FIR, and 6-stage halfband IIR filters

6. CONCLUSION

In this paper, we evaluated time domain performance of different decimation filter architectures that can be used in high resolution $\Delta\Sigma$ ADC. Ten filters were designed based on the typical anti-aliasing 64 times decimation filter design specifications. The group delay properties of both linear phase and non-linear phase multistage filters were investigated in consideration with other frequency performances such as passband, stopband and transition band.

The analysis showed that the multistage minimum phase FIR filter and multistage polyphase IIR filter are promising for low group delay audio applications. The group delay increases near the Nyquist frequency, but this might not be a problem for some live audio applications.

The theoretical implementation costs were listed. However, these results were just for typical reference designs. There are vast amount of methods and techniques being developed in optimization of filter design and realization, such as the optimal minimum phase FIR filter design method [22][23]. For halfband FIR filter design, minimum phase filter design without altering the linear impulse response [24][25] could be interesting to consider. It would be interesting for authors to further research some of these specific areas.

Simulated SNR for typical architectures were evaluated as well. But in real hardware implementations, the effects of quantization of coefficients needs to be further investigated. There are also other practical factors such as hardware and software architectures, which might influence the tradeoff and selection of decimation filters.

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